

and the points $P(t)$ take up the whole curve (C), e.g. between $t = 0$ and ∞ . Then the relations between the given S_p become *non-linear* inequalities, well known for the problem of moments.

REFERENCES

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AN INEQUALITY FOR MILL'S RATIO

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Mr. R. D. Gordon¹ recently proved the inequalities

$$\frac{x}{x^2 + 1} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \leq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2} dt \leq \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad \text{for } x > 0.$$

In the present note we show that the lower inequality can be replaced by the better estimate

$$\frac{\sqrt{4 + x^2} - x}{2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \leq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2} dt.$$

PROOF: According to a well-known theorem of Jensen², for $f(t)$ convex and $g(t) \geq 0$ in the interval (a, b) , the following inequality holds

$$f \left[\frac{\int_a^b t g(t) dt}{\int_a^b g(t) dt} \right] \leq \frac{\int_a^b f(t) g(t) dt}{\int_a^b g(t) dt}.$$

For $a = x$, $b = \infty$, $f(t) = 1/t$, $g(t) = te^{-\frac{1}{2}t^2}$, this inequality gives

$$\int_x^\infty te^{-\frac{1}{2}t^2} dt / \int_x^\infty t^2 e^{-\frac{1}{2}t^2} dt \leq \int_x^\infty e^{-\frac{1}{2}t^2} dt / \int_x^\infty te^{-\frac{1}{2}t^2} dt.$$

Since

$$\int_x^\infty te^{-\frac{1}{2}t^2} dt = e^{-\frac{1}{2}x^2} \quad \text{and} \quad \int_x^\infty t^2 e^{-\frac{1}{2}t^2} dt = xe^{-\frac{1}{2}x^2} + \int_x^\infty e^{-\frac{1}{2}t^2} dt,$$

¹ R. D. Gordon, "Values of Mill's ratio of area to bounding ordinate of the normal probability integral for large values of the argument," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 364-366.

² See for example: G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, Cambridge, 1934, p. 150-151.