THE PROGENY OF AN ENTIRE POPULATION[1]

By Alfred J. Lotka

Metropolitan Life Insurance Company

The literature on renewal theory has grown to considerable dimensions, until even admittedly incomplete bibliographies list over 100 titles. But a surprisingly small proportion of these publications exhibits any practical applications to concrete data, and such applications as have been made (e.g. by Wicksell, Hadwiger, Rhodes) are for the most part of restricted scope.

Anyone who has been following the development will, I think, feel that this is unfortunate. It has a double disadvantage. On the one hand the purely theoretical discussions emphasize difficulties which in practice may be relatively unimportant, being inherent either in some of the unrealistic *ad hoc* examples discussed, or in the expressions used to fit smooth curves to the basic data, rather than in these data themselves. On the other hand some real difficulties in application to actual data seem to require further clarification.

Several of the applications that have been made, including some of my own, are restricted to following up the "progeny" of a "population element" comprising only individuals all originating at the same time and therefore all of the same age (in the case of industrial equipment installation all made at one point of time). The analysis set forth in the treatment of this special case is competent also to deal with the practically more important case of the progeny of an initial population of given age distribution, though no example of this has hitherto been published.[2] Such an example will now be given, and at the same time this will afford an opportunity to clarify some points in the presentation of the more general case.

Let N_t be the total number of females at time t, and $c_t(a)$ the number comprised within the age limits a and $a + da$. Also, let $m_t(a)$ be the age-specific fertility of females of age a, counting daughters only. If α and ω are, respectively the lower and the upper limit of the female reproductive period, and $B(t)$ the annual births of females, then

$$B(t) = \int_\alpha^\omega N_t c_t(a) m_t(a) \, da. \tag{1}$$

However, it is not in this perfectly general form that the relation is to be applied. The case to be considered is that in which the "initial" population is throughout its "future" development, subject to constant age-specific fertility

[1] Compare A. J. Lotka, "The progeny of a population element," *Am. Jour. Hygiene*, Vol. 8 (1928), p. 875.

[2] An example was given by the writer in an oral communication to the Eighth American Scientific Congress, May 1940, the Proceedings of which have not so far been published.