ON MUTUALLY FAVORABLE EVENTS

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Introduction. For a set of arbitrary events, E. J. Gumbel, M. Fréchet and the author[1] have recently obtained inequalities between sums of certain probability functions. One of the results of the author is the following:

Let $E_1, \cdots, E_n$ be n arbitrary events and let $p_m(\nu_1, \cdots, \nu_k)$ denote the probability of the occurrence of at least m events out of the k events $E_{\nu_1}, \cdots, E_{\nu_k}$. Then, for $k = 1, \cdots, n - 1$ and $1 \leqq m \leqq k$ we have

$$\binom{n-m}{k-m} \Sigma p_m(\nu_1, \cdots, \nu_{k+1}) \leqq \binom{n-m}{k-m+1} \Sigma p_m(\nu_1, \cdots, \nu_k),$$

where the summations extend respectively to all combinations of $k + 1$ and k indices out of the n indices $1, \cdots, n$.

In course of proof of the above inequalities it appears that similar inequalities between products instead of sums can be obtained under certain assumptions regarding the nature of interdependence of the events. We shall first study the nature of such assumptions, and then proceed to the proof of the said inequalities (Theorems 1 and 2). It may be noted that the inductive method used here serves equally well for the proof of the inequalities cited above, though somewhat longer, but apparently our former method is not applicable here.

That events satisfying our assumptions actually exist, is shown by an application to the elementary theory of numbers. The author feels incompetent to discuss other possible fields of application.

1. Let a set of events be given

$$E_1, E_2, \cdots, E_n, \cdots$$

and let E'_i denote the event non-E_i. Let $p(i)$ denote the probability of the occurrence of E_i, $p(i')$ that of the occurrence of E'_i. For convenience we assume that for any i $p_i(1 - p_i) \neq 0$; events with the exceptional probabilities 0 or 1 may evidently be left out of account.

Let $p(\nu_1 \cdots \nu_k)$ denote the probability of the occurrence of the conjunction $E_{\nu_1} \cdots E_{\nu_k}$ and let $p(\mu_1 \cdots \mu_h, \nu_1 \cdots \nu_k)$ denote the probability of the occurrence of $E_{\nu_1} \cdots E_{\nu_k}$, on the hypothesis that $E_{\mu_1} \cdots E_{\mu_h}$ have occurred. The μ's or ν's may be accented.

Definition 1: *If $p(\nu_1, \nu_2) > p(\nu_2)$, we say that the occurrence of the event E_{ν_1} is favorable to the occurrence of the event E_{ν_2}, or simply that E_{ν_1} is favorable to E_{ν_2}.*

[1] "On the probability of the occurrence of at least m events among n arbitrary events," *Annals of Math. Stat.* Vol. 12 (1941), pp. 328–338.