LIMITED TYPE OF PRIMARY PROBABILITY DISTRIBUTION APPLIED TO ANNUAL MAXIMUM FLOOD FLOWS

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1. Theoretical statement of problem. There is no doubt that Gumbel's recent paper "The Return Period of Flood Flows" [1] has supplied an admirably simple technique for engineers to use in approximating the trend of *return periods* of annual maximum flood flows for purposes of extrapolation. This treatment is scientifically of great interest because it introduces for the first time into a subject already treated at considerable length by engineers, the theory of the probability distribution of maximum values as developed by Fisher and Tippett, von Mises, and others.[1] However, certain further observations should be made concerning the approach used by Gumbel.

Let x represent the measure of daily stream flow having a probability distribution $w(x)$. Let the probability distribution of the associated annual maximum stream flows be denoted by $V(x)$ with

$$W(x) = \int_0^x V(s)\, ds, \tag{1}$$

denoting probability that annual maxima be less than or equal to x. The *return period* $T(x)$ of an annual maximum flow of measure x is then defined by

$$T(x) = \frac{1}{1 - W(x)}. \tag{2}$$

In this paper the probability distribution $w(x)$ will be called the *primary* probability distribution associated with the probability distribution of maximum values $V(x)$ and its *cumulative* distribution $W(x)$.

Gumbel argues that for the type of primary probability distribution that might reasonably be expected to apply, $W(x)$ will be of the type introduced by R. A. Fisher:

$$W(x) = \exp\,[-\exp - \alpha(x - u)]. \tag{3}$$

It is further implied that a primary probability distribution involving an upper limit would lead to a probability distribution of maximum values of the type

$$W_1(x) = \frac{k}{u}\left(\frac{u}{x}\right)^{k+1} \cdot e^{-(u/x)^k}, \tag{4}$$

for which moments of order k or higher do not exist. The inference is then drawn that a primary probability distribution leading to such a cumulative distribution of maximum values would seem to be less likely to be the correct

[1] See references at end of Gumbel's paper, loc. cit.