ON THE PROBLEM OF MULTIPLE MATCHING

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1. Introduction. The problem of determining the distribution of the number of "hits" or "matchings" under random matching of two decks of cards has received attention from a number of authors within the last few years. In 1934 Chapman [2] considered pairings between two series of t elements each, and later [3] generalized the problem to series of u and $t(\leq u)$ elements respectively. In the same paper he also considered the distribution of the mean number of correct matchings resulting from n independent trials, and gave a method, and tables, for determining the significance of any obtained mean. In 1937 Bartlett [1] considered matchings of two decks of cards, using a number of interesting moment generating functions. In 1937 Huntington [12, 13] gave tables of probabilities for matchings between decks with the compositions (5^5), (4^4), and (3^3), where (s^t) denotes a deck consisting of s of each of t kinds of cards. More generally $(s_1s_2 \cdots s_t)$ denotes s_1 cards of the first kind, s_2 of the second, etc. Sterne [16] has given the first four moments of the frequency distribution for the (5^5) case and has fitted a Pearson Type I distribution function to the distribution. Sterne obtained his results by considering the probabilities in a 5×5 contingency table. He also considered the 4×4 and 3×3 cases. In 1938 Greville [7] gave a table of the exact probabilities for matchings between two decks of compositions (5^5). Greenwood [4] obtained the variance of the distribution of hits for matchings between two decks having the respective compositions (s^t) and $(s_1s_2 \cdots s_t)$ with $s_1 + s_2 + \cdots + s_t = st = n$, and where it is not necessary that all the s's should be different from zero. Earlier Wilks [19] had considered the same problem for $t = 5$ and $n = 25$.

In a very interesting paper Olds [15] in 1938 used permanents to express a moment generating function suitable for the problem in question. He obtained factorial moments and the first four ordinary moments about the mean, first for two decks with composition (4^2), and then for two decks of composition (s^t). In 1938 Stevens [17] considered a contingency table in connection with matchings between two sets of n objects each, and gave the means, variances, and covariances of the single cell entries and various sub-totals of the cell entries. Stevens [18] also gave a treatment of the problem of matchings between two decks which was based on elementary considerations. In 1940 Greenwood [6] gave the first four moments of the distribution of hits between two decks of any composition whatever, generalizing the problem which had been treated earlier by Olds [15]. Finally in 1941, Greville [8] gave the exact distribution of hits for matchings between two decks of arbitrary composition. He also considered the problem from the standpoint of a contingency table, as had been done earlier by Stevens.