ON THE POWER FUNCTION OF THE ANALYSIS OF VARIANCE TEST[1]

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It is known[2] that the general problem of the analysis of variance can be reduced by an orthogonal transformation to the following canonical form: Let the variates $y_1, \cdots, y_p, z_1, \cdots, z_n$ be independently and normally distributed with a common unknown variance σ^2. The mean values of $z_1, \cdots, z_n$ are known to be zero, and the mean values $\eta_1, \cdots, \eta_p$ of the variates $y_1, \cdots, y_p$ are unknown. The canonical form of the analysis of variance test is the test of the hypothesis that

$$\eta_1 = \eta_2 = \cdots = \eta_r = 0 \qquad (r \leq p) \tag{1}$$

where a single observation is made on each of the variates $y_1, \cdots, y_p, z_1, \cdots, z_n$.

In the theory of the analysis of variance the test of the hypothesis (1) is based on the critical region

$$\frac{y_1^2 + \cdots + y_r^2}{z_1^2 + \cdots + z_n^2} \geq c \tag{2}$$

where the constant c is chosen so that the size of the critical region is equal to the level of significance α we wish to have. The critical region (2) is identical with the critical region

$$\frac{y_1^2 + \cdots + y_r^2}{y_1^2 + \cdots + y_r^2 + z_1^2 + \cdots + z_n^2} \geq c' = \frac{c}{c+1}. \tag{3}$$

It is known that the power function of the critical region (3) depends only on the single parameter

$$\lambda = \frac{1}{\sigma^2} \sum_{i=1}^{r} \eta_i^2. \tag{4}$$

Denote the power function of the critical region (3) by $\beta_0(\lambda)$. P. L. Hsu has proved[3] the following optimum property of the region (3): *Let W be a critical region which satisfies the following two conditions*:

(a) *The size of W is equal to the size of the region* (3).

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[1] Presented at a joint meeting of the Institute of Mathematical Statistics and the American Mathematical Society in New York, December, 1941.

[2] See for instance P. C. Tang, "The power function of the analysis of variance tests," *Stat. Res. Mem.*, Vol. 2, 1938.

[3] P. L. Hsu, "Analysis of variance from the power function standpoint," *Biometrika*, January, 1941.