

ON THE RATIO OF THE VARIANCES OF TWO NORMAL POPULATIONS

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1. Introduction and summary. Suppose that we have two samples E_1 and E_2 from normal populations π_1 and π_2 with unknown means and variances. Let us designate by θ the ratio of the variance of π_1 to that of π_2 . *The two problems discussed in this paper are to formulate in terms of E_1 and E_2 , and to compare,*

(i) *significance tests for the hypothesis that the unknown ratio θ is equal to a given positive number θ_0 , and*

(ii) *confidence intervals for θ .*

Since, on the one hand, these problems are of considerable importance to the practical statistician and the teacher of statistics, and on the other, they cry for the application of recently developed theory which is unfortunately not yet familiar to many practical workers and teachers, the development has been divided into two parts: Part I, it is hoped, will be intelligible to the above class of readers; part II, slanted toward a smaller circle, is more esoteric, general, and condensed.

More specifically, in part I it is pointed out that any choice of limits on the *F*-distribution satisfying the condition that the sum of the areas in the tails be equal to a prescribed number, leads to solutions of problems (i) and (ii). After considering and then ruling out the "one-sided" situations in which it is appropriate to use only one tail, two conditions are proposed (*ad hoc* and on an intuitive basis) for the "two-sided" case,—a symmetry condition, and a condition for logarithmically shortest confidence intervals. The second condition leads to a choice of limits on the *F*-distribution. From other considerations,—