$-\infty \leq v \leq \infty$. Thus neither y nor ω has an asymptotic normal distribution. It is, of course, this fact which makes the criterion of minimum variance illusory.

3. Other polynomial distribution functions. Let repeated samples of n independent values of x be drawn from a population characterized by $D(x) = \frac{k+1}{a^{k+1}} x^k$, $0 \leq x \leq a$, and k a positive integer or zero. It can be shown that the best linear estimate of the mean of the population is $y = \frac{(k+1)n+1}{n(k+2)} x_n$, where as before x_n is the largest item of the sample. The sampling distribution of y is easily obtained. It follows that

$$\sigma_y^2 = \frac{(k+1)a^2}{(k+2)^2[(k+1)n^2+2n]} = \frac{k+3}{n(k+1)+2} \sigma_{\bar{x}}^2,$$

where as usual $\bar{x}$ is the arithmetic mean of the sample. Again, if we write $u = \left(y - \frac{k+1}{k+2} a\right) \Big/ \sigma_y$, the limit of the distribution of u as n approaches infinity is, as before, e^{u-1}, $-\infty \leq u \leq 1$.

A NOTE ON TOLERANCE LIMITS

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Among various statistical problems arising in the process of controlling quality in mass production, a rather important one appears to be the determination of tolerance limits when the variability of the product is known to be due to random factors. This problem was recently treated in a pioneer article by Wilks. This note will point out a relationship between tolerance limits and confidence limits (used in the sense of Neyman), and will use this concept to establish tolerance limits when the product is described by two qualities, the measurements on which are assumed to have a bivariate normal distribution.

For the case of a single variate, the problem of finding tolerance limits as stated by Wilks is to find a sample size n, and two functions $L_1(x_1 \cdots x_n)$ and $L_2(x_1 x_2 \cdots x_n)$ so that if $P = \int_{L_1}^{L_2} f(x)\, dx$ denotes the conditional probability of a future observation falling between the random variates L_2 and L_1, then

$$E(P) = \alpha, \quad \text{and Prob.} \quad [\alpha - \Delta_1 \leq P \leq \alpha + \Delta_2] \geq \beta.$$

The relationship between confidence limits and tolerance limits will arise if confidence limits are determined, not for a parameter of the distribution, but for

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