

equal. Now

$$E(y_{i_1}^{\alpha_1} y_{i_2}^{\alpha_2} \cdots y_{i_k}^{\alpha_k}) = E(y_{i_1} y_{i_2} \cdots y_{i_k})$$

is the probability that  $y_{i_1} = 1, y_{i_2} = 1, \cdots, y_{i_k} = 1$ , simultaneously. This probability is either zero (for example, when  $|i_2 - i_1| = 1, |i_3 - i_2| = 1$ , etc. or when  $i_1 = n$ , etc.) or  $\left(\frac{2}{n}\right)^k + O\left(\frac{1}{n^{k+1}}\right)$ . Moreover, the ratio of the number of  $k$ -tuples  $i_1, i_2, \cdots, i_k$  for which the probability is zero to the number of  $k$ -tuples for which the probability is  $\left(\frac{2}{n}\right)^k + O\left(\frac{1}{n^{k+1}}\right)$  is  $O\left(\frac{1}{n}\right)$ . Let  $Z_i (i = 1, \cdots, n)$  be independent stochastic variables each with the same distribution such that the probability that  $Z_i = 1$  is  $2/n$  and the probability that  $Z_i = 0$  is  $(n - 2)/n$ . It follows readily that the limit, as  $n \rightarrow \infty$ , of the  $j$ th moment ( $j = 1, 2, \cdots$ , ad inf.) of  $y$  about the origin, is the same as the limit of the same moment of  $Z$ , where

$$Z = \sum_{i=1}^n Z_i.$$

Since the  $Z_i$  are independently distributed, and since each can take only the values 0 and 1, the probability of the value 1 being  $2/n$ , the  $j$ th moment of  $Z$  about the origin approaches, as  $n \rightarrow \infty$ ,

$$\mu_j = e^{-2} \sum_{i=1}^{\infty} \frac{i^j 2^i}{i!},$$

which is the  $j$ th moment about the origin of the Poisson distribution with mean value 2. By the preceding paragraph,  $\mu_j$  is also the limit of the  $j$ th moment of  $y$  about the origin. Now von Mises [2] has proved that if the  $j$ th moment ( $j = 1, 2, \cdots$ , ad inf.) of a chance variable  $X_n$ , ( $n = 1, 2, \cdots$ , ad inf.), approaches, as  $n \rightarrow \infty$ , the  $j$ th moment of a Poisson distribution, then the distribution of  $X_n$  approaches the Poisson distribution with corresponding mean value. From this it follows that  $y$  has in the limit the distribution (1). We have already shown that  $y$  and  $n - W(R)$  have the same limiting distribution, so that the required result follows.

#### REFERENCES

- [1] J. WOLFOWITZ, *Annals of Math. Stat.*, Vol. 13 (1942), p. 247.  
 [2] R. v. MISES, *Zeitschrift für die angewandte Math. und Mechanik*, Vol. 1 (1921), p. 298.

### NOTE ON CONSISTENCY OF A PROPOSED TEST FOR THE PROBLEM OF TWO SAMPLES

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Certain tests for the hypothesis that two samples are from the same population assume nothing about the distribution function except that it is continuous. Since the power functions of these tests have not been obtained, optimum