A MATRIX PRESENTATION OF LEAST SQUARES AND CORRELATION THEORY WITH MATRIX JUSTIFICATION OF IMPROVED METHODS OF SOLUTION

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1. Introduction and summary. It is the aim of this paper to exhibit, by using elementary matrix theory, the basic concepts of least squares and correlation theory, the solution of the normal equations, and the presentation and justification of recently developed and newly proposed techniques into a single, compact, and short presentation. We shall be mainly concerned with the following topics:

a. Basic least squares theory including derivation of normal equations, the theoretical solution of these equations (regression coefficients), the standard errors of these solutions, and the standard error of estimate.

b. The more specific theory (correlation theory) resulting from applying the general least squares results to the standardized distributions.

c. A matrix presentation of the Doolittle solution.

d. A simple matrix justification of methods, previously presented, for getting least squares and multiple correlation constants from the entries of an abbreviated Doolittle solution.

e. A presentation of a more general theory which the matrix presentation reveals.

f. The outline of a "square root" method as an alternative to theDoolittle method.

The reader should be familiar with elementary matrix theory such as that outlined on pages 1–57 of Aitken's book [1].

No previous knowledge of the Doolittle technique is demanded although a familiarity with the notation and contents of two earlier papers [2], [3] is advised, particularly for those who are interested in the computational aspects.

The presentation here is theoretical and is not concerned with such computational topics as the number of decimal places required, etc. With reference to the number of places, the reader is referred to the recent paper of Professor Hotelling [4].

2. Notation. Let $[x'_{ij}]$ with $1 \leq i \leq N$ and $1 \leq j \leq n$ be the n by N matrix of observed variates of n "predicting variables" for N individuals with i indicating the individual and j the variable. Let $[y'_i]$ be the one by N column matrix of the observed variates of the "predicted" variable. Let the matrices of deviations from the variable means be indicated by $[x_{ij}] = X$ and $[y_i] = Y$. Then by the least squares hypothesis we are to find numbers $b_{y1\cdots}, b_{y2\cdots}, \cdots, b_{yn\cdots}$ such that

$$e_i = y_i - (x_{i1}b_{y1\cdots} + x_{i2}b_{y2\cdots} + \cdots + x_{in}b_{yn\cdots}),$$

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