

ON THE THEORY OF SYSTEMATIC SAMPLING, I

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1. Introduction. It is no longer necessary to demonstrate a need for the theory of designing samples. Many of the policy and operating decisions of both government and private industry are based on samples. There has been an increasing tendency in government and industry to make use of sampling theory.³

Unfortunately there are still considerable differences between the theory and practice of sampling. The origins of these differences are, on the one hand, the ignorance of administrators concerning the practical contributions that sampling theory can make, and on the other, the lack of sampling theory permitting the evaluation of some useful sampling designs.

Much has been and is being done towards bringing theory and practice into agreement.⁴ Administrators and samplers are each successfully educating the others. However, there still exist sampling designs for which an adequate theory has not been developed, even though experience indicates that if such a theory were developed it would demonstrate the superiority of those designs over others for which a theory has been developed.

Perhaps the major omission of sampling theory today is the lack of any statistical method for reaching a decision on whether to take a completely random sample of n elements of a population of N elements, or to take a systematic sample, that is, to begin with element i , and select elements $i, i + k, \dots, i + (n - 1)k$, as the sample, the starting point i being chosen at random and $N = kn$ approximately.⁵ It is with respect to this question of whether to take a systematic⁶ or random sample that the statistician is in a dilemma because he has the alternative of recommending a systematic sampling procedure for which no theory exists, or a random sampling procedure that may well yield worse

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³ The recognition of the need for statisticians who know sampling theory has resulted in courses in sampling being given in some of the colleges and universities.

⁴ One need only refer to the recent development of positions, the duties of which include giving advice on sampling techniques as well as working in the field of application.

⁵ In this paper we will assume that $N = kn$. To do away with that assumption would not add much in the way of generality while it would require some fairly detailed discussion. It may be remarked that when N is not exactly kn , then systematic sampling procedures in which all starting points have equal probability of selection are biased, although the bias is usually trivial. If N is known this bias can be removed by sampling proportionate to possible size of systematic sample.

⁶ As we define systematic sampling procedures, a systematic sampling procedure is a random sampling procedure in which many of the C_n^N selections of n from N items are excluded.