SOME IMPROVEMENTS IN WEIGHING AND OTHER EXPERIMENTAL TECHNIQUES

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When several quantities are to be ascertained there is frequently an opportunity to increase the accuracy and reduce the cost by combining suitably in one experiment what might ordinarily be considered separate operations. The theory of design of experiments developed as a branch of modern mathematical statistics, and of which fundamental considerations are set forth in R. A. Fisher's book [1], provides many improvements of this kind. Since the main interests of Fisher and other originators of this theory have been in biology, the applications so far made have been chiefly biological in character, excepting for certain economic and social investigations involving stratified sampling. The possibilities of improvement of physical and chemical investigations through designed experiments based on the theory of statistical inference have scarcely begun to be explored.

The following example is due to F. Yates [2]. A chemist has seven light objects to weigh, and the scale also requires a zero correction, so that eight weighings are necessary. The standard error of each weighing is denoted by $\sigma$, the variance therefore by $\sigma^2$. Since the weight assigned to each object by customary techniques is the difference between the reading of the scale when carrying that object and when empty, the variance of the assigned weight is $2\sigma^2$, and its standard error is $\sigma\sqrt{2}$.

The improved technique suggested by Yates consists of weighing all seven objects together, and also weighing them in groups of three so chosen that each object is weighed four times altogether, twice with any other object and twice without it. Calling the readings from the scale $y_1, \ldots, y_7$ we then have as equations for determining the unknown weights $a, b, \ldots, g$,

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\begin{align*}
  a + b + c + d + e + f + g &= y_1 \\
  a + b + c &= y_2 \\
  a + d + e &= y_3 \\
  a + f + g &= y_4 \\
  b + d + f &= y_5 \\
  b + e + g &= y_6 \\
  c + d + e + f &= y_7.
\end{align*}
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1 Presented at the Wellesley meeting of the Institute of Mathematical Statistics, Aug. 13, 1944.

297