INVERSE TABLES OF PROBABILITIES OF ERRORS OF THE SECOND KIND

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1. Introduction. The problem of testing linear hypotheses was discussed by Kolodziejczyk [1], and later in greater detail by Tang [2], who computed a table giving the probabilities of errors of the second kind P_{II} for a range of values of the two degrees of freedom f_1 and f_2, ($f_1 = 1(1)8, f_2 = 2, 4, 6(1)30, 60, \infty$)[1] and for two fixed levels $P_I = .01$ and $.05$ of the probability of errors of the first kind. These tables are in terms of a parameter φ, ($\varphi = 1(.5)3(1)8$) whose statistical significance, or rather that of

$$\lambda = (f_1 + 1)\varphi^2/2$$

is discussed in Tang's paper. A restatement of the problem of testing linear hypotheses in a more canonical form, giving an interpretation of λ, will also be found in a recent paper by Wald [3].

Professor Neyman has felt for some time that a table giving $\varphi = \varphi(a, b, \alpha, \beta)$ as a function of the two degrees of freedom $f_1 = 2a$, and $f_2 = 2b$, and of the two probability levels $\alpha = P_I$ and $\beta = 1 - P_{II}$ would be more useful for statistical purposes, where β is the probability of detecting the falsehood of the hypothesis tested. A paper by Professor Neyman explaining this point of view and giving applications of the present tables to some statistical problems will appear shortly. These tables were computed in the Statistical Laboratory of the University of California,[2] and give values of φ for the following range of parameters:

$$(\alpha, \beta) = (.01, .7), (.01, .8), (.05, .7), (.05, .8)$$

$$f_1 = 1(1)10, 12, 15, 20, 24, 30, 40, 60, 80, 120, \infty.$$

$$f_2 = 2(2)20, 24, 30, 40, 60, 80, 120, 240, \infty.$$

2. Analytic definitions. The statistical parameter

$$\lambda = \lambda(a, b, \alpha, \beta) = (a + \tfrac{1}{2})\, \varphi^2(a, b, \alpha, \beta) \tag{1}$$

can be thought of as an inverse function connected with the hypergeometric distribution. Inverse functions $y(\alpha)$, $u(a, \alpha)$ and $x(a, b, \alpha)$ of the better known normal, Gamma and Beta distributions respectively have all been tabulated,

[1] The notation $m = r(s)t$ is equivalent to $m = r, r + s, r + 2s, \ldots, t$.

[2] These tables were begun by Miss Leone Gintzler, and were carried on by Mark Eudy under a University of California Research Grant. The bulk of the computing was done, however, by the author and by Mrs. Julia Robinson under a grant of the American Philosophical Society.