NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

A COMBINATORIAL FORMULA AND ITS APPLICATION TO THE THEORY OF PROBABILITY OF ARBITRARY EVENTS[1]

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An important principle, known as a proposition in formal logic or the method of cross-classification can be stated as follows.[1]

Let F and f be any two functions of combinations out of $(\nu) = (1, 2, \cdots, n)$. Then the two formulas

$$F((\alpha)) = \sum_{(\beta) \in (\nu)-(\alpha)} f((\alpha) + (\beta)) \tag{1.1}$$

$$f((\alpha)) = \sum_{(\beta) \in (\nu)-(\alpha)} (-1)^b F((\alpha) + (\beta)) \tag{2.1}$$

are equivalent.

As an immediate application to the theory of probability of arbitrary events, we have the set of inversion formulas[2]

$$p((\alpha)) = \sum_{(\beta) \in (\nu)-(\alpha)} p[(\alpha) + (\beta)] \tag{3.1}$$

$$p[(\alpha)] = \sum_{(\beta) \in (\nu)-(\alpha)} (-1)^b p((\alpha) + (\beta)) \tag{4.1}$$

where $p((\alpha))$ is the probability of the occurrence of at least $E_{\alpha_1}, E_{\alpha_2}, \cdots, E_{\alpha_a}$ out of n arbitrary events $E_1, E_2, \cdots, E_n$ and $p[(\alpha)]$ is the probability of the occurrence of $E\alpha_1, E\alpha_2, \cdots, E\alpha_a$ and no others among the n events, $(\alpha_1, \alpha_2, \cdots, \alpha_a)$ denoting a combination of the integers $(1, 2, \cdots, n)$. They can be made to play a central rôle in the theory, since they supply a method for converting the fundamental systems of probabilities, $p[(\alpha)]$ and $p((\alpha))$, one into the other.

We may further generalize (1.1) and (2.1) by considering combinations with repetitions. Let such a combination be written as

$$(\alpha) = (\alpha^r)^- = (\alpha_1^{r_1}\alpha_2^{r_2} \cdots \alpha_a^{r_a})$$

[1] For the notations and definitions see K. L. Chung, "On fundamental systems of probabilities of a finite number of events," *Annals of Math. Stat.*, Vol. 14 (1943), pp. 123–133.

[2] Cf. Fréchet, *Les probabilités associées à un système d'événements compatibles et dépendants*, Hermann, Paris (1939), formulas (55) and (58).