RANDOM WALK IN THE PRESENCE OF ABSORBING BARRIERS

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1. Introduction. The problem of random walk (along a straight line) in the presence of absorbing barriers can be stated as follows:

A particle, starting at the origin, moves in such a way that its displacements in consecutive time intervals, each of duration Δt, can be represented by independent random variables

$$X_1, X_2, X_3, \cdots$$

Moreover, if at some time the total (cumulative) displacement becomes $>p$ ($p \geq 0$) or $< -q$ ($q \geq 0$) the particle gets absorbed. The problem is to determine the probability that "the length of life" of the particle is greater than a given number t. This problem also admits an interpretation in terms of a game of chance in which the player quits when he loses more than q or wins more than p. An interesting paper on this type of problem by A. Wald[1] appeared recently in the *Annals*. Wald assumes that the X's are identically distributed and that their mean and standard deviation are different from 0.[2] He is then mostly interested in the limiting case when both the mean and the standard deviation become small. The object of this paper is to propose a different method of attack which in some cases leads to an answer in closed form. The method we use has been employed repeatedly in statistical mechanics in the study of the so called order-disorder problem. It is due, I believe, to E. W. Montroll[3]. As far as the author knows this method was never used in connection with the classical probability theory and this seems to furnish an additional reason for publishing this paper.

2. The simplest discrete case. We assume that each X is capable of assuming the values 1 and -1 each with probability $\frac{1}{2}$, and for simplicity sake we let $\Delta t = 1$. Note that, unlike in Wald's case, the mean of X is 0. Denote by N the random variable which represents the "length of life" of the particle and let (m an integer)

$$\delta(m) = \begin{matrix} \frac{1}{2} & m = 1 \quad \text{or} \quad m = -1, \\ 0 & \text{otherwise.} \end{matrix}$$

[1] A. Wald "On cumulative sums of random variables," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 283–296.

[2] Since this was written Professor Wald informed the author that he can easily avoid the condition that the mean should be zero.

[3] See for instance E. W. Montroll, "Statistical Mechanics of nearest neighbor systems," *Jour. of Chem. Physics*, Vol. 9 (1941), pp. 706–721.

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