NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

A NOTE CONCERNING HOTELLING'S METHOD OF INVERTING A PARTITIONED MATRIX

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Professor Hotelling recently presented several methods of computing the inverse of a matrix.[1] Among these was a method of partitioning a square matrix of $2p$ rows into four square matrices, a, b, c and d, of p rows each, resulting in the partitioned matrix,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

The inverse of this matrix can also be written as a partitioned matrix,

$$\begin{bmatrix} A & C \\ B & D \end{bmatrix}.$$

Then, multiplying the original matrix by its inverse we get four matrix equations,

$$\begin{aligned} aA + bB &= 1 \qquad & aC + bD &= 0 \\ cA + dB &= 0 \qquad & cC + dD &= 1. \end{aligned}$$

These equations can be solved for A, B, C, and D.

Professor Hotelling's solution requires the inversion of four p-rowed matrices. It is possible, however, to solve these equations by formulas involving only two inversions. The formulas are

$$\begin{aligned} D &= (d - ca^{-1}b)^{-1} \qquad & B &= -Dca^{-1} \\ C &= -a^{-1}bD \qquad & A &= a^{-1} - a^{-1}bB. \end{aligned}$$

As an example of the procedure let the given matrix be

$$\left[\begin{array}{cc:cc} 26 & -10 & 15 & 32 \\ 19 & 45 & -14 & -8 \\ \hdashline -12 & 16 & 27 & 13 \\ 32 & 29 & -35 & 28 \end{array}\right].$$

[1] Harold Hotelling. "Some new methods of matrix calculation," *Annals of Math. Stat.*, Vol. 14 (1943), pp. 1–34.