ON THE DESIGN OF EXPERIMENTS FOR WEIGHING AND MAKING OTHER TYPES OF MEASUREMENTS

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1. Introduction. In a recent paper, Hotelling [1] has discussed the basic principles of the theory of the design of efficient experiments for estimating the true unknown weights of p given objects by means of a specified number N of weighings, $p \leq N$ in case the scale is free from bias and $p \leq N - 1$ if it has a bias the unknown value of which has to be estimated from the same data. He has emphasized the importance of these designs in other kinds of measurements besides weighing of objects and has called attention to the need for further mathematical research for obtaining a "comprehensive general solution." Such a solution has now been obtained in case the number of weighings N is at our choice. Some other general designs have also been given in this paper for specified values of N and p.

2. Estimation of unknown weights and efficiency of a design. Using Hotelling's notation, we may write

$$E(y_\alpha) = \sum_{i=1}^{p} x_{i\alpha} b_i \tag{1}$$

where $i = 1, 2, \cdots p$, on the assumption that there is either zero bias in the scale or the bias is known *a priori*, and $\alpha = 1, 2, \cdots N$. $E(y_\alpha)$ is the expectation of the αth weighing. For a biassed scale, we may take $i = 0, 1, 2, \cdots p$. The efficient estimate of each of the b_i's has been derived by Hotelling by the method of least squares. It is of interest to obtain these estimates by the use of the theory of linear estimation as developed by Bose [2] and Rao [3].

Assuming that $y_1, y_2, \cdots y_N$ are N stochastic variates forming a multivariate normal system with the variance and covariance matrix given by

$$u = [u_{ij}], \tag{2}$$

it follows from Rao's generalization of Markoff's theorem that the best unbiassed estimates of the b_i's are given by the solutions of the normal equations

$$X'U^{-1}XB' = X'U^{-1}Y', \tag{3}$$

where $B = [b_1 b_2 \cdots b_p]$ and $Y = [y_1 y_2 \cdots y_N]$, and B' and Y' denote as usual the transpose of the row vectors B and Y, i.e. column vectors.

In the present case, the assumption is that all the N stochastic variates are uncorrelated and have a common variance σ^2, so that

$$U^{-1} = \frac{1}{\sigma^2} I . \tag{4}$$