This is a general result, applicable to any arrangement of the terms of an arbitrary square matrix A, subject only to the conditions that $| A | \neq 0$ and that no diagonal term of A is zero. In this latter exceptional case, the iterative method itself obviously cannot be applied.

The criterion (11) clearly shows that the order in which the elements of the matrix A are arranged is important. For instance, it is plain that an arrangement in which the diagonal terms are large and the off-diagonal terms, particularly the post-diagonal terms, are small will tend to favor convergence.

A somewhat relaxed condition, which is sufficient but not necessary, is obtained through the use of an inequality used by Hotelling[3], namely,

$$N(B^m) \leq [N(B)]^m, \tag{12}$$

in which $N(B)$ is the norm of the matrix B, that is, the square root of the sum of the products of its elements by their complex conjugates, or in the case of a real matrix the square root of the sum of the squares of the elements.

The condition is that, if

$$N(A_1^{-1}A_2) < 1, \tag{13}$$

then

$$\lim_{m \to \infty} (A_1^{-1}A_2)^m = 0. \tag{14}$$

Criterion (13) is readily computed, since A_1^{-1}, the reciprocal of a triangular matrix is readily computed, and the post-multiplication by A_2 involves a number of zero terms.

A more stringent condition than (13) though still not a necessary condition, is that if some finite number p can be found such that

$$N(A_1^{-1}A_2)^p < 1, \tag{15}$$

then (14) follows. Since n matrix squarings result in a value of $p = 2^n$, the size of the norm for fairly large values of p can be investigated without excessive labor.

A REMARK ON INDEPENDENCE OF LINEAR AND QUADRATIC FORMS INVOLVING INDEPENDENT GAUSSIAN VARIABLES

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The purpose of this note is to call attention to the following useful theorem, which to the best of my knowledge was never stated explicitly.

If $X_1, X_2, X_3, \cdots X_n$ are identically distributed, independent Gaussian random variables each having mean 0, *then the necessary and sufficient condition that*

$$\sum_{j,k=1}^{n} a_{jk} X_j X_k \quad \textit{and} \quad \sum_{j=1}^{n} \alpha_j X_j = \alpha \cdot X$$