AN INEQUALITY FOR DEVIATIONS FROM MEDIANS

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In a recent note in these *Annals*, Birnbaum and Zuckerman [1] proved that if:

(1) $X_1, X_2, \cdots, X_n$ are independent random variables with the same distribution (i.e., form a sample),
(2) their common distribution is symmetric about zero,

then

$$E(|X_1 + X_2 + \cdots + X_n|) \geq \varphi(n) \cdot E(|X_1|),$$

where

$$\varphi(2k+1) = \varphi(2k+2) = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2k+1)}{1 \cdot 2 \cdot 4 \cdot 6 \cdots (2k)}.$$

It is the purpose of the present note to extend this to the following, more general, result:

Theorem. *If*

(i) $X_1, X_2, \cdots, X_n$ *are independent random variables,*
(ii) *the median of each* X_i *is zero,*

then

$$E(|X_1 + X_2 + \cdots + X_n|) \geq \frac{\varphi(n)}{n} E(|X_1| + |X_2| + \cdots + |X_n|).$$

It will be convenient to let $d_i = E(|X_i|)$ and

$$\bar{d} = \frac{1}{n} \sum d_i = \frac{1}{n} E(|X_1| + |X_2| + \cdots + |X_n|),$$

so that the desired inequality becomes

$$E(|X_1 + X_2 + \cdots + X_n|) \geq \varphi(n) \cdot \bar{d}.$$

Define e_i by

$$e_i = \int_0^\infty x dF_i(x),$$

where $F_i(x)$ is the cumulative distribution function of X_i. Since

$$d_i = E(|X_i|) = -\int_{-\infty}^0 x dF_i(x) + \int_0^\infty x dF_i(x),$$