ON THE USE OF THE SAMPLE RANGE IN AN ANALOGUE OF STUDENT'S *t*-TEST

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Let $x_1, \cdots, x_N$ represent independent observations on a variate x which is normally distributed with mean μ and variance σ^2. Assuming no prior information about the value of either parameter, let H_0 be the hypothesis that μ is equal to or less than a specified quantity μ_0. The classical test of this asymmetrical form of "Student's" hypothesis [1] is based upon the statistic

$$t = \sqrt{N}(\bar{x} - \mu_0) \Big/ \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}},$$

the region of rejection being defined by the relation $t > t_\epsilon$.

For certain applications of a routine nature, however, such as production line inspection, the usefulness of this test is rather seriously impaired by the arithmetical work involved in the computation of t. For this reason Dodge [2] and Knudsen [3] among others have proposed tests of H_0 based on a statistic of the form

$$G = \frac{\bar{x} - \mu_0}{w}$$

where w is the sample range. It is the object of this note to show how the probability distribution of G can be obtained with the aid of the distribution law of w tabulated by Pearson and Hartley [4], and to present some numerical results which indicate that the power of the resulting test is the same for all practical purposes as that of "Student's" t-test for sample sizes $N \leq 10$.

The calculation of the percent points of the G distribution is greatly facilitated by the following result, which does not appear to be generally known:

Lemma: *If $\bar{x}$ and w represent respectively the average and the range of a sample of N independent observations on a normally distributed variate x, then $\bar{x}$ and w are statistically independent.*

Proof: No generality is lost by putting $\mu = 0$, $\sigma^2 = 1$. The joint characteristic function of $\bar{x}$ and the $\frac{1}{2}N(N - 1)$ differences $x_j - x_k$, $(j < k)$, is then

$$\varphi(t, t_{jk}) = (2\pi)^{-(N/2)} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sum_j x_j^2 + i\frac{t}{N}\sum_j x_j + i\sum_{j,k} t_{jk}(x_j - x_k)}\, dx_1 \cdots dx_N$$

where the summation runs from 1 to N on each index with the understanding that $t_{jk} \equiv 0$ for $j \geq k$. The usual process of completing the square in the exponent then yields

$$\varphi(t, t_{jk}) = e^{-\frac{1}{2}\sum_j \left[\frac{t}{N} + \sum_k (t_{jk} - t_{kj})\right]^2} \cdot (2\pi)^{-(N/2)} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sum_j \left\{x_j - i\left[\frac{t}{N} + \sum_k (t_{jk} - t_{kj})\right]\right\}^2} dx_1 \cdots dx_N.$$