**4. Sampling from** $F(x) = \int_0^x He^{-Ht}\,dt$, $(0 \leq x < \infty; H > 0)$. If $F(x) = \int_0^x He^{-Ht}\,dt$, the probability function of $S$ can be determined but is very cumbersome in the form in which it is known to the writer. The characteristic function, say $g(\theta)$, of the probability function of $S$ will be given instead. By use of (2.1) it can be shown that:

$$g(\theta) = e^{iD\theta} \prod_{\lambda=1}^{n-1} \left\{ \frac{i\theta e^{D(i\theta - \lambda H)} - \lambda H}{i\theta - \lambda H} \right\}, \tag{4.1}$$

where $i = \sqrt{-1}$.

The expected value, $E(S)$, and variance, $\sigma_S^2$, of $S$ are:

$$\begin{aligned} E(S) &= D + \frac{1}{H} \sum_{\lambda=1}^{n-1} \frac{(1 - e^{-DH\lambda})}{\lambda}, \\ \sigma_S^2 &= \frac{1}{H^2} \sum_{\lambda=1}^{n-1} \frac{(1 - e^{-2DH\lambda})}{\lambda^2} - \frac{2D}{H} \sum_{\lambda=1}^{n-1} \frac{e^{-DH\lambda}}{\lambda}. \end{aligned} \tag{4.2}$$

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# INFORMATION GIVEN BY ODD MOMENTS


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The widespread use of the third moment about the mean as a measure of skewness and the belief engendered by this use that a distribution is symmetric if its third moment is zero prompt the question of how much information about a distribution can be deduced from a knowledge of its odd moments. An answer to this question is: *Let $F(x)$, a cumulative distribution function; $\{\mu_{2n-1}\}$, $(n = 1, 2, \cdots)$, a sequence of real numbers; and $\epsilon > 0$ be arbitrary. There exists a c.d.f., $F^*(x)$, having as odd moments the terms of the given sequence and such that*

$$| F(x) - F^*(x) | \leq \epsilon, \ \underline{all}\ x. \tag{1}$$

If the mean of $F(x)$ is equal to $\mu_1$ and the variance of $F(x)$ is not zero, it can be shown that $F^*(x)$ may be chosen so that in addition the variance of $F^*(x)$ is equal to that of $F(x)$.

An immediate consequence of our statement is that a distribution need not be