or returning to the original notation and retaining terms in $1/N$,

$$r \sim r_\infty\left(1 + \frac{1}{2N}\right). \tag{3}$$

If $x_p$ is defined by $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x_p} e^{-t^2/2}\,dt = p$ we know from [3] that

$$\frac{\chi_\beta^2}{n} \sim 1 + \frac{\sqrt{2}\,x_{1-\beta}}{\sqrt{n}} + \frac{2}{3}\,\frac{x_{1-\beta}^2 - 1}{n}. \tag{4}$$

Proceeding formally and retaining terms in $1/N$ we obtain

$$\left(\frac{n}{\chi_\beta^2}\right)^{\frac{1}{2}} = \left(1 - \frac{x_{1-\beta}}{\sqrt{2N}} + \frac{4 + 5x_{1-\beta}^2}{12N}\right)$$

and multiplying by the expression for $r$ given by equation (3) we find the desired expansion for $\lambda$.

$$\lambda \sim r_\infty\left(1 - \frac{x_{1-\beta}}{\sqrt{2N}} + \frac{5x_{1-\beta}^2 + 10}{12N}\right). \tag{5}$$

Recall that both $r_\infty$ and $x_{1-\beta}$ are readily obtainable from tables of the normal curve; in fact, $r_\infty$ is defined by

$\frac{1}{\sqrt{2\pi}}\int_{-r_\infty}^{r_\infty} e^{-t^2/2}\,dt = \gamma$ and $x_{1-\beta}$ is defined by $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x_{1-\beta}} e^{-t^2/2}\,dt = 1 - \beta$.

A comparative table of approximate and exact values of $\lambda$ is given in Table 1. From the table we see that for $N \geqq 800$ the error is less than 1 in the 4th significant figure, and for $N \geqq 160$ the error is less than 1 in the 3rd significant figure within the limits of $\beta$ and $\gamma$ considered. The approximation will be less exact for higher values of $\beta$ and $\gamma$.

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# THE PROBABILITY DISTRIBUTION OF THE MEASURE OF A RANDOM LINEAR SET

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**1. Introduction.** Consider a random sample $O_n(x_1, \cdots, x_n)$ of $n$ values of a one-dimensional random variable $x$ having cumulative distribution function $F(x)$. Let there be associated with each $x$ an interval of length $D$ centered at $x$