APPROXIMATE FORMULAS FOR THE PERCENTAGE POINTS AND NORMALIZATION OF t AND χ^2 [1]

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1. Introduction. The χ^2 Distribution and Student's t-distribution are functions of a parameter n (degrees of freedom) and approach the normal distribution as n approaches infinity. The normal distribution is a good approximation to these distributions for large n. For small or moderate n, a better approximation may be obtained by using a function of t(or χ^2) which approaches the normal distribution more rapidly as n increases. Hotelling and Frankel [7] pointed out that an additional advantage of the normalization of a distribution is that further statistical tests are possible with the normalized variate. Normalizing t(or χ^2) is equivalent to transforming it into a function which is normally distributed to a required degree of approximation; that is, a normally distributed variate of zero mean and unit variance is expressed as a function of t(or χ^2) in powers of $1/n$.

The reverse problem of expressing t(or χ^2) as a function of a normally distributed variate of zero mean and unit variance in powers of $1/n$ is also of practical importance in connection with significance tests for which the significance levels, or percentage points, of the t and χ^2 distributions are required.

Cornish and Fisher [1] (see also [2]) have given a method for the normalization of distributions which approach normality as the number of degrees of freedom, n, increases and whose cumulants are expressed in power series of $1/n$, so that the order of magnitude of the rth cumulant is that of $n^{-(r-1)}$. A method has also been given for expressing a variate with such a distribution as a function of a normally distributed variate of zero mean and unit variance in powers of $1/n$.

It is the purpose of this note to apply the Cornish-Fisher method (1) to the derivation of asymptotic formulas for the percentage points of the t and χ^2 distributions and (2) to the normalization of these distributions. Tables are given which indicate the accuracy of these approximations and compare them with other approximations. Tables are also given to facilitate the calculation of the approximations for the percentage points of t and χ^2.

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[2] Henry Goldberg died April 19, 1945.