ABSTRACTS OF PAPERS

Presented on August 21, 1946, at the Cornell meeting of the Institute

1. **A Test of Randomness in Two Dimensions.** HOWARD LEVENE, Columbia University.

A square of side N is divided into N^2 unit cells, and each cell takes on the characteristics A or B with probabilities p and $q = 1 - p$ respectively, independently of the other cells. A cell is an "upper left corner" if it is A and the cell above and cell to the left are not A. Let V_1 be the total number of upper left corners and let V_2, V_3, V_4 be the number of similarly defined upper right, lower right, and lower left corners respectively. Let $V = (V_1 + V_2 + V_3 + V_4)/4$. It is proved that V is normally distributed in the limit with $E(V) = p(Nq + p)^2$ and $\sigma^2(V) \sim N^2pq^2(4 - 20p + 45p^2 - 27p^3)/4$. The conditional limit distribution of V when p is estimated from the data, and the limit distribution of a related quadratic form are also obtained. These statistics are in a sense a generalization of the run statistics used for testing randomness in one dimension.

2. **Asymptotic Distribution of Moments from a System of Linear Stochastic Difference Equations.** HERMAN RUBIN, Cowles Commission for Research in Economics.

Let $\sum_{\tau=0}^{\infty} B_\tau y'_{t-\tau} + \Gamma z'_t = u'_t$, $(t = 1, 2, \cdots)$, be a complete system of linear stochastic difference equations determining y_{ti} (the coordinates of y_t), $t > 0$, in terms of y_{ti}, $t \leq 0$, and z_{tk} (the coordinates of z_t), which are assumed to be fixed variates, and the random variables u_{ti} (the coordinates of u_t). Such a system is called a stable if for every bounded set of fixed variates, and $E(u'_t u_t)$ uniformly bounded, $E(y'_t y_t)$ is uniformly bounded. This condition is shown to be equivalent to $\sum | h_{ij\tau} |$ finite, where $y'_t = \sum_{\tau=0}^{\infty} H_\tau(u'_{t-\tau} - \Gamma z'_{t-\tau}) + \sum_{\nu=0}^{\infty} J_{t,\nu} y'_{-\nu}$ is the solution of the above difference equation. Let Q_t be an infinite quadratic form in $y_{t-\tau,i}$ and $z_{t-\nu,k}$ $(\tau, \nu = 0, 1, \cdots)$ with coefficients depending only on i, k, τ, and ν. Such a quadratic form is called convergent if the sum of the absolute values of the coefficients is finite. It is shown under fairly general conditions that the mean of a convergent quadratic form is asymptotically normally distributed with variance $0\left(\frac{1}{T}\right)$.

3. **Conditional Expectation and Unbiased Sequential Estimation.** DAVID BLACKWELL, Howard University.

It is shown that $E[f(x_\alpha)E_\alpha y] = E(fy)$ whenever $E(fy)$ is finite, and that $\sigma^2(E_\alpha y) \leq \sigma^2(y)$, with equality holding only if $E_\alpha y = y$, where $E_\alpha y$ denotes the conditional expectation of y with respect to the family of chance variables x_α. These results imply that whenever there is a sufficient statistic u and an unbiased estimate t, not a function of u only, for a parameter p, the function $E_u t$, which is a function of u only, is an unbiased estimate for p with variance smaller than that of t. A sequential unbiased estimate for a parameter is obtained, such that when the sequential test terminates after i observations, the estimate is a function of a sufficient statistic for the parameter with respect to these observations. A special case of this estimate is that obtained by Girshick, Mosteller, and Savage (*Annals of Math. Stat.*, Vol. XVII (1946), pp. 13–23) for the parameter of a binomial distribtion.

4. **A Discussion of the Ehrenfest Model.** Preliminary report. MARK KAC, Cornell University.

A particle moves along a straight line in steps Δ, the duration of each step being τ. The probabilities that the particle at $k\Delta$ will move to the right or left are $(1/2)(1 - k/R)$