examination of the proof of Theorem 3 shows that it would go through with little change if equation (3) were replaced by the requirement that $|m(\alpha)|$ be bounded. We therefore obtain the following result: If for a doubly simple region there exists an unbiased estimate $p(\alpha)$ of p, not identically equal to $\hat{p}(\alpha)$, then not only is $p(\alpha)$ not proper, but also, no matter how large M, there exists a boundary point α such that $|p(\alpha)| > M$. The uselessness of such an estimate is manifest.

The author is of the opinion that freedom from bias is not necessarily an indispensable characteristic of an optimum estimate. In general there is no reason for requiring the first moment of the estimate rather than any other moment to be the unknown parameter. The justification in any particular case must be based on special conditions of the problem.

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DIFFERENTIATION UNDER THE EXPECTATION SIGN IN THE FUNDAMENTAL IDENTITY OF SEQUENTIAL ANALYSIS

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1. Introduction. Let $\{z_\alpha\}$ ($\alpha = 1, 2, \cdots$, ad inf.) be a sequence of random variables which are independently distributed with identical distributions. Let a be a positive, and b a negative constant. For each positive integral value m, let Z_m denote the sum $z_1 + \cdots + z_m$. Denote by n the smallest integral value for which Z_n does not lie in the open interval (b, a). For any random variable u, let the symbol $E(u)$ denote the expected value of u. The following identity, which plays a fundamental role in sequential analysis, has been proved in [1].

$$E[e^{Z_n t}\varphi(t)^{-n}] = 1, \tag{1.1}$$

where

$$\varphi(t) = E(e^{zt}) \tag{1.2}$$

and the distribution of z is equal to the common distribution of $z_1, z_2, \cdots$, etc. Identity (1.1) holds for all points t in the complex plane for which $\varphi(t)$ exists and $|\varphi(t)| \geq 1$.