NOTES

This section is devoted to brief research and expository articles on methodology and other short items.

ON SEQUENTIAL BINOMIAL ESTIMATION

By J. Wolfowitz

Columbia University

The present note, written after a reading of the very interesting paper by Girshick, Mosteller, and Savage [1], is for the purpose of adding a few remarks in the nature of a supplement. For the sake of brevity the notation and terminology of [1] are adopted in toto.

Theorem 1 below generalizes Theorem 1 of [1]. In Theorem 2′ we formulate explicitly the fact which lies at the basis of the GSM method of estimation. Parts of the proofs of Theorems 3 and 4 of [1] are simply proofs of special cases of this (e.g., equation (2) of [1]). We then use this fact repeatedly in proving Theorem 3, which states that the Girshick-Mosteller-Savage estimate is the only proper unbiased estimate for sequential tests defined by regions which we shall call doubly simple.

A doubly simple region is defined precisely below. Intuitively we may describe such a region as the one between two curves $y = f_1(x)$ and $x = f_2(y)$, where $f_1(x)$ is defined and monotonically non-decreasing for all non-negative x, $f_2(y)$ is defined and monotonically non-decreasing for all non-negative y, $f_1(0) > 0$, $f_2(0) > 0$. If the two curves intersect, the region is finite, and the values of the functions f_1 and f_2 beyond the point of intersection are of no interest. This description is of course purely heuristic, because in actual fact only integral values of the variables come into play, and intersection of the curves, for example, is not needed to make the region finite. Since the question of finite regions is completely settled by [1], Theorem 7, only non-finite regions remain to be discussed, and the precise definition given below is such as to imply that the region is not finite. It seems to the present writer that at least many of the non-finite sequential tests which may be developed for meaningful statistical problems will require doubly simple regions. The Wald sequential binomial test [2] defines such a region, which also falls within the scope of Theorem 6 of [1]. It is easy to see that there exist closed regions which are doubly simple and do not satisfy the conditions of this theorem.

By a "proper" estimate $p(\alpha)$ we shall mean an estimate such that $0 \leq p(\alpha) \leq 1$ for every α. It is difficult to see how any estimate which is not proper can make much sense.

Theorem 1. *A sufficient condition that a region R be closed is that* $\liminf_{n \to \infty} \frac{A(n)}{\sqrt{n}} < \infty$, *where $A(n)$ is the number of accessible points of index n.*