Therefore

$$S_{2p+1}(\alpha, \alpha) = 0. \tag{18}$$

When $n$ is an integer, either $n + 1$ or $n + 2$ is odd. Therefore when (15) holds, one of either (7) or (8) will be satisfied identically if we take $\beta = \alpha$. The other may then be solved for $\alpha$.

As an example, suppose one had the moments $\mu_0 = 1$, $\mu_1 = \frac{1}{2}$, $\mu_2 = \frac{7}{24}$, $\mu_3 = \frac{3}{16}$, $\mu_4 = \frac{31}{240}$, and wished to obtain an $f(x)$ such that $f(0) = 0, f(1) = 0$. In this case $n = 2$, and (15) is satisfied. It follows that (7) is satisfied identically when $\beta = \alpha$, and (8) gives

$$\frac{\Gamma(2\alpha + 5)}{\Gamma(\alpha + 1)} + 4\,\frac{\Gamma(2\alpha + 6)}{\Gamma(\alpha + 2)}\left(-\frac{1}{2}\right) + 6\,\frac{\Gamma(2\alpha + 7)}{\Gamma(\alpha + 3)}\left(\frac{7}{24}\right) + 4\,\frac{\Gamma(2\alpha + 8)}{\Gamma(\alpha + 4)}\left(-\frac{3}{16}\right) + \frac{\Gamma(2\alpha + 9)}{\Gamma(\alpha + 5)}\left(\frac{31}{240}\right) = 0.$$

This easily reduces to

$$1 - 4\,\frac{\alpha + 5/2}{\alpha + 1} + 7\,\frac{(\alpha + 5/2)(\alpha + 3)}{(\alpha + 1)(\alpha + 2)} - 6\,\frac{(\alpha + 5/2)(\alpha + 7/2)}{(\alpha + 1)(\alpha + 2)} + \frac{31}{240}\,\frac{(\alpha + 5/2)(\alpha + 7/2)}{(\alpha + 1)(\alpha + 2)} = 0,$$

which reduces to the quadratic

$$4\alpha^2 - 6\alpha + 5 = 0,$$

from which

$$\alpha = \beta = 3/4 \pm (1/4)\sqrt{11}\,i. \tag{19}$$

These may be substituted into (4)–(6) to complete the solution.

## REFERENCE

[1] G. Szegö, *Orthogonal Polynomials*, Amer. Math. Soc. Colloquium Pub., No. 23, 1939.

# CONSISTENCY OF SEQUENTIAL BINOMIAL ESTIMATES

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The notion of consistency of an estimate, introduced by R. A. Fisher, applies to a sequence of estimates which converge stochastically, with boundlessly increasing sample size, to the parameter (or parameters) being estimated. Each estimate is a function of a sample of observations, the number in each sample being determined independently of the observations themselves. In sequential estimation, on the other hand, the number of observations is itself a chance