

$0 < p_1 < 0.1$ , the limiting value of  $p_i$  will be zero; if  $p_1 > 0.1$ , the limiting value will be 0.9. The interesting point is that if the initial probability is in the neighborhood of 0.1, an infinitesimal change in its value may produce a finite change in the stable limiting probabilities; and that for the initial probability equal 0.1 one would have an unstable equilibrium of the system. This consideration shows why it is important to know how the probability  $p_i$  converges towards a certain point. As we have previously shown, the points of convergence are roots of the eq.  $p = f(p)$  but there roots which are not points of convergence.

Similar reasoning could be applied to more complicated systems belonging to our general scheme of contagion. Consequently, the most important result is not that the considered assembly may have a probability tending to some value in the range  $0 \leq p \leq 1$ , but that under certain conditions the limiting probability may jump from one value to another by changing the initial probability by an arbitrarily small amount.

## REFERENCES

- [1] F. EGGENBERGER AND G. POYLA, *Zeits. fur Ang. Math. und Mech.* Vol. 3 (1923), p. 279.
- [2] M. GREENWOOD AND G. U. YULE, *Roy. Stat. Soc. Jour.*, Vol. 83 (1920), p. 255.
- [3] R. LUDERS, *Biometrika*, Vol. 26 (1934), p. 108.
- [4] J. NEYMAN, *Annals of Math. Stat.*, Vol. 10 (1939), p. 35.
- [5] W. FELLER, *Annals of Math. Stat.*, Vol. 14 (1943) p. 389.
- [6] F. CERNUSCHI AND E. SALEME, *Anales Soc. Cientifica Argentina*, Vol. 138 (1944), p. 201.

---

## FITTING CURVES WITH ZERO OR INFINITE END POINTS

BY EDMUND PINNEY

*Oregon State College*

The problem of determining a suitable equation to fit an empirically determined curve over a given interval has been of great importance in statistical work, in experimental science, and in engineering technology. Since infinitely many types of equations may be made to fit the data with required accuracy, the choice of a "suitable" type of equation depends on the qualitative nature of the empirical curve, on the use to which the equation is to be put, and upon considerations of simplicity.

As a function type, the polynomial has, because of its simplicity, been enormously useful. The function type studied here is a little more general than the polynomial type, being particularly useful in the case of empirical curves that become zero or infinity at one or both ends of the interval.

Without loss of generality the interval in which the equation is to fit the curve may be taken as  $0 \leq x \leq 1$ . It is assumed that, by numerical means or otherwise, a finite set of moment  $\mu_m = \int_0^1 yx^m dx$  may be computed,  $y$  being the ordinate of the empirical curve.