NOTES

This section is devoted to brief research and expository articles on methodology and other short items.

ON THE STUDENTIZATION OF SEVERAL VARIANCES

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1. Introduction. In a recent paper [1] the author considered the problem of eliminating several variances simultaneously from probability statements concerning the mean of a normally distributed variable. The general situation envisaged was as follows. We supposed that we had an observed quantity y which could be assumed to be normally distributed about a population mean η with variance $\sigma_y^2 = \sum_{i=1}^{k} \lambda_i \sigma_i^2$, where the λ_i are known positive numbers and the σ_i^2 unknown population variances. It was supposed further that the data provided estimates s_i^2 of the σ_i^2 based on f_i degrees of freedom, and having the sampling distributions

$$(1) \qquad p(s_i^2)\, ds_i^2 = \frac{1}{\Gamma(\frac{1}{2}f_i)} \exp\left\{-\tfrac{1}{2}\frac{f_i s_i^2}{\sigma_i^2}\right\} \left(\tfrac{1}{2}\frac{f_i s_i^2}{\sigma_i^2}\right)^{\frac{1}{2}f_i-1} d\left(\tfrac{1}{2}\frac{f_i s_i^2}{\sigma_i^2}\right)$$

and that these estimates were distributed independently of each other and of y. The problem was to make statements about the magnitude of the difference $y - \eta$ which would involve explicitly only the observed variances s_i^2. The probability of the truth of the statements was also to be entirely independent of the population values σ_i^2.

The solution was given implicitly in a formal mathematical expression and a general process of developing successive terms in a series expansion was described. In the present communication a slightly different way of reaching this development is provided.

2. General method. If the f_i are large enough the ratio

$$(2) \qquad v = \frac{y - \eta}{\sqrt{\Sigma \lambda_i s_i^2}}$$

can be taken to be normally distributed with mean zero and standard deviation unity. This suggests that, when the f_i are not necessarily large, we might approach the matter by seeking some other function

$$(3) \qquad x = g\{s_1^2, s_2^2, \cdots, s_k^2, y - \eta\}$$

which will still be normally distributed with the same mean and standard deviation. We shall see that such a function can be found, although the method to be followed leads us first to another expression