A STATISTICAL PROBLEM CONNECTED WITH THE COUNTING OF RADIOACTIVE PARTICLES

By Sten Malmquist

Institute of Statistics, University of Upsala, Sweden

1. Introduction. Our problem refers to random events forming a sequence in time or in space, *e.g.* particles emitted by a radioactive matter. By omitting certain elements of the given sequence, say f, we form another sequence, say g. The rule of omission involves an arbitrarily prescribed constant u. The rule to be followed in forming g is:

Case I: Let a be an element in f and g. The next element to be included in g is then the first element in f which follows a after a distance greater than u.

Case II: Let a be an element in f and g. The next element to be included in g is then the first element in f which follows a at a distance greater than u from the preceding element in f, whether this belongs to g or not.

When the events are represented by impulses emitted by a radioactive matter and feeding a recorder with a constant resolving time u, the new sequence consists of the counted impulses. The two cases correspond to the reaction of different types of recorders. The distinction between the two transformations has caused some confusion. It has, however, been clearly pointed out by Ruark and Brammer [5].

v. Bortkiewicz [2] seems to be the first who has considered problems related to the transformed sequence. Starting from investigations by Rutherford, Geiger, and others, concerning the number of recorded α-particles during a certain interval of time, say T, he observed that the distribution of this number was similar to that of Poisson but with a slightly smaller dispersion. This fact he supposed to be caused by a constant resolving time u of the recorder. By means of certain assumptions he tried to calculate the effect on the mean and the dispersion by the transformation in Case I, supposing the cumulative distribution function $F(t)$ for the distance between two consecutive elements in the sequence f is given by

$$F(t) = 1 - e^{-at},$$

where here and in what follows, t denotes a non-negative variable.

Considering Case II with $F(t)$ as above, Levert and Scheen [4] have recently worked out an expression for the distribution of the number of elements during T in the sequence g.

Gnedenko [3] has considered the distribution of the number of lost elements in Case I with particular regard to the initial state of rest.

Alaoglu and Smith [1] considered problems referring to successive transformations of a sequence. When, for example, a sequence of particles enters a tube-counter and amplifier, together acting with a resolving time u_1, and