corresponding value of R_3 computed for a chosen p, then approximately, the proportion p' of plotted errors should fall within the circle of radius R_3.

REFERENCES

[1] HENRY SCHEFFÉ, *Armor and Ordinance Report No. A-224*, OSRD No. 1918, Div. 2, pp. 60–61.

[2] S. S. WILKS, *Mathematical Statistics*, Princeton Univ. Press, 1943, p. 131.

A NOTE ON THE EFFICIENCY OF THE WALD SEQUENTIAL TEST

BY EDWARD PAULSON

Institute of Statistics, University of North Carolina

The sequential likelihood ratio test of Wald for testing the hypothesis H_0 that the probability density function is $f(X, \theta_0)$ against the one-sided alternative H_1 that the function is $f(X, \theta_1)$ has been shown [1] to have the optimum property of minimizing the expected number of observations at the two points $\theta = \theta_0$ and $\theta = \theta_1$. Tables showing the actual magnitude of the percentage saving of this sequential procedure compared with the classical "best" non-sequential test have been calculated (see [1], page 147) for the normal case when

$$f(X, \theta) = \frac{1}{\sqrt{2\pi}} \exp \frac{-(X - \theta)^2}{2}.$$

In this note we will show that when θ_1 is close to θ_0, the percentage saving is independent of the particular function $f(X, \theta)$ and the particular values θ_1 and θ_0, so that the tables mentioned above can be used to show the percentage saving for any one-sided sequential test involving a single parameter, provided $f(X, \theta)$ satisfies some weak restrictions.

Let $f(X, \theta)$ be the probability density function of a random variable. Let $E_i(n)$ denote the expected value (when $\theta = \theta_i$) of the number of independent observations required by the Wald sequential procedure to test the hypothesis H_0 that $\theta = \theta_0$ against $\theta = \theta_1 = \theta_0 + \Delta$ with probabilities α of rejecting H_0 when $\theta = \theta_0$ and β of accepting H_0 when $\theta = \theta_1$. Let N be the number of independent observations required to achieve the same probabilities α and β for testing the hypothesis $\theta = \theta_0$ against $\theta = \theta_1$ by the most powerful non-sequential test. Let U_α and U_β be defined by the relations

$$\alpha = \frac{1}{\sqrt{2\pi}} \int_{U_\alpha}^{\infty} \exp\left\{-\frac{t^2}{2}\right\} dt$$

and

$$\beta = \frac{1}{\sqrt{2\pi}} \int_{U_\beta}^{\infty} \exp\left\{-\frac{t^2}{2}\right\} dt.$$