ON THE ASYMPTOTIC DISTRIBUTION OF DIFFERENTIABLE STATISTICAL FUNCTIONS

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Introduction. If n real variables $x_1, x_2, \cdots, x_n$ are subject to a probability distribution with the element $dV_1(x_1)dV_2(x_2) \cdots dV_n(x_n)$ one can ask for the distribution of any function f of $x_1, x_2, \cdots x_n$. We are primarily interested in *statistical functions*, i.e. in functions that depend on the *repartition*[1] $S_n(x)$ of the n quantities $x_1, x_2, \cdots x_n$ only. The simplest case is that of the *linear statistical functions*

$$f = \int \psi(x)\, dS_n(x) = \frac{1}{n}[\psi(x_1) + \psi(x_2) + \cdots + \psi(x_n)]. \tag{1}$$

The so-called Central Limit Theorem of Probability Calculus states that the distribution of a linear statistical function, if n tends to infinity, approaches more and more the normal (Gauss) distribution if some very general conditions linking $\psi(x)$ and the $V_\nu(x)$ are fulfilled. It has been shown, ten years ago, [2] that the restriction to linear functions here is immaterial. Much more general

[1] The function $S_n(x)$ is called the repartition of the real quantities $x_1, x_2, \cdots, x_n$ if $nS_n(x)$ is the number of those among the $x_1, x_2, \cdots, x_n$ that are smaller than or equal to x.