

ON THE ASYMPTOTIC DISTRIBUTION OF DIFFERENTIABLE STATISTICAL FUNCTIONS

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Introduction. If n real variables x_1, x_2, \dots, x_n are subject to a probability distribution with the element $dV_1(x_1)dV_2(x_2) \cdots dV_n(x_n)$ one can ask for the distribution of any function f of x_1, x_2, \dots, x_n . We are primarily interested in *statistical functions*, i.e. in functions that depend on the *repartition*¹ $S_n(x)$ of the n quantities x_1, x_2, \dots, x_n only. The simplest case is that of the *linear statistical functions*

$$(1) \quad f = \int \psi(x) dS_n(x) = \frac{1}{n} [\psi(x_1) + \psi(x_2) + \cdots + \psi(x_n)].$$

The so-called Central Limit Theorem of Probability Calculus states that the distribution of a linear statistical function, if n tends to infinity, approaches more and more the normal (Gauss) distribution if some very general conditions linking $\psi(x)$ and the $V_i(x)$ are fulfilled. It has been shown, ten years ago, [2] that the restriction to linear functions here is immaterial. Much more general

¹ The function $S_n(x)$ is called the repartition of the real quantities x_1, x_2, \dots, x_n if $nS_n(x)$ is the number of those among the x_1, x_2, \dots, x_n that are smaller than or equal to x .