Write $y = \log_e \nu_r$. Then we have

$$\nu_r \frac{dy}{dr} = \int_{-\infty}^{\infty} |x|^r \log_e |x| \, dF(x),$$

$$\nu_r^2 \frac{d^2 y}{dr^2} = \int_{-\infty}^{\infty} |x|^r \, dF(x) \cdot \int_{-\infty}^{\infty} |x|^r \log_e^2 |x| \, dF(x) - \left\{ \int_{-\infty}^{\infty} |x|^r \log_e |x| \, dF(x) \right\}^2$$

$\geqq 0$, by Schwarz's inequality.

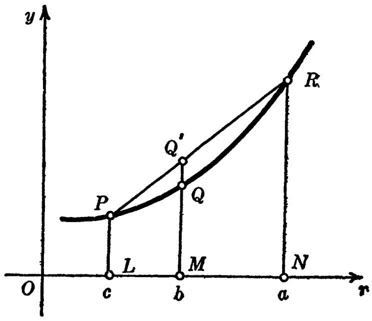


FIG. 1

It follows that the function $y$ is convex (or exceptionally a straight line), and, on referring to the figure, it appears that

(1) $$MQ \leq MQ'$$

for all chords $PR$. If the abscissae of the points $L, M, N$ are $c, b, a$, respectively, where $c \leqq b \leqq a$, the inequality (1) leads at once to the relation

$$\log_e \nu_b \leqq \frac{a-b}{a-c} \log_e \nu_c + \frac{b-c}{a-c} \log_e \nu_a .$$

Hence

$$\nu_b^{a-c} \leqq \nu_c^{a-b} \nu_a^{b-c},$$

which is the usual form of the Liapounoff Inequality.

## REMARK ON THE NOTE "A GENERALIZATION OF WARING'S FORMULA"

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Before submitting for publication the note "A generalization of Waring's formula," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 218–219 the author made a diligent effort to ascertain, through correspondence with mathematicians and