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A NOTE ON THE FUNDAMENTAL IDENTITY OF SEQUENTIAL ANALYSIS

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1. Introduction. Let $\{z_i\}$, $(i = 1, 2, 3, \cdots)$, be a sequence of real valued random variables identically distributed according to the cumulative distribution function $F(z)$. Define the sums $Z_N = z_1 + z_2 + \cdots + z_N$ for every positive integer N. Choose two positive constants a and b and define the random variable n as the smallest integer N for which one of the inequalities $Z_N \geqq a$ or $Z_N \leqq -b$ holds. The notations $P(u \mid F)$ and $E(u \mid F)$ will denote the probability of u and its expectation respectively assuming that F is the distribution of the z_i.

Wald [1] has established the results contained in the following lemmas.

Lemma 1. *If the variance of $F(z)$ is positive, $P(n < \infty \mid F)$ equals one.*

Lemma 2. *If there exists a positive number δ such that $P(e^z < 1 - \delta \mid F) > 0$ and $P(e^z > 1 + \delta \mid F) > 0$ and if the moment generating function $\varphi(t) = E(e^{zt} \mid F)$ exists for all real values of t, then $\varphi(t)$ has one and only one minimum at some finite value $t = t_0$. Moreover, $\varphi''(t) > 0$ for all real values of t.*

It is the purpose of this note to establish the following extension of the validity of certain results given by Wald [1], [2].

Theorem.[1] *Under the conditions of Lemma 2 the identity*

$$E\{e^{Z_n t}[\varphi(t)]^{-n} \mid F\} = 1 \tag{1}$$

[1] Wald's results show (1) to be valid for all complex t in the domain over which $|\varphi(t)| \geqq 1$ and the validity of the differentiation clause for all real t in that domain. The importance of the present extension arises from the fact that, if $E(x \mid F) \neq 0$, then $0 < \varphi(t) < 1$ on a certain interval of the real axis.