ON THE CHARLIER TYPE B SERIES

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1. Introduction. The Type B series of Charlier has been discussed in some detail in the literature (See references at the end of the paper). The problem of the convergence of the Type B series has been considered by Pollaczek-Geiringer [12], [13], Szegö [12] (page 110), Uspensky [16], Jacob [5], Schmidt [16] and Obrechkoff [11]. There is presented in the following a method of development of the Type B series which is believed to be of some interest, including a necessary and sufficient condition for the convergence which is basically the same as that of Schmidt [16]. A result of Steffensen [17] is extended and shown to be related to the Charlier Type B series.

2. Statement of results. Consider the function $p(r)$, defined for $r = 0, 1, 2, \cdots$, and such that

$$\sum_{r=0}^{\infty} p(r) = 1; \qquad \sum_{r=0}^{\infty} | p(r) | = A \tag{2.1}$$

where A is some finite value. Let the n-th factorial moment be defined by

$$\begin{aligned} \mu_{(0)} &= 1 \\ \mu_{(n)} &= \sum_{r=0}^{\infty} r(r-1)(r-2)\cdots(r-n+1)p(r), \qquad (n = 1, 2, \cdots) \end{aligned} \tag{2.2}$$

For arbitrary λ let

$$\begin{aligned} L_n = \mu_{(n)} - n\mu_{(n-1)}\lambda + \frac{n(n-1)}{2!}\mu_{(n-2)}\lambda^2 \\ - \frac{n(n-1)(n-2)}{3!}\mu_{(n-3)}\lambda^3 + \cdots + (-1)^n\lambda^n. \end{aligned} \tag{2.3}$$

We prove the following results:

Theorem. *A necessary and sufficient condition that the function* $p(r)$ *of* (2.1) *may be expressed by the absolutely convergent series*

$$p(r) = \frac{e^{-\lambda}\lambda^r}{r!} + L_1 \frac{\partial}{\partial\lambda}\frac{e^{-\lambda}\lambda^r}{r!} + \frac{L_2}{2!}\frac{\partial^2}{\partial\lambda^2}\frac{e^{-\lambda}\lambda^r}{r!} + \cdots \tag{2.4}$$

is that

$$1 + |\mu_{(1)}| + \tfrac{1}{2!}|\mu_{(2)}| + \tfrac{1}{3!}|\mu_{(3)}| + \cdots + \frac{1}{n!}|\mu_{(n)}| + \cdots \tag{2.5}$$

converges where L_n *is defined as in* (2.3).