

mal). The numbers would have equal probabilities insofar as this is attainable by chaining. To obtain a random three-digit decimal series it would be necessary to reject the numbers above 999 (decimal). This would amount to only 2.34% of the available data. As before, rejection could be accomplished easily in the binary series by use of a ten-stage electronic counter.

Several promising devices are being considered for tabulating random numbers in accordance with the principles discussed herein. Electronic or electrical systems actuated by cosmic rays seem to be the most desirable. Tabulating equipment may be wired to turn out random numbers, possibly as a by-product of other card runs.

If only a few random numbers are needed, they can be obtained by much simpler methods. For example, a coin may be tossed, letting heads and tails represent +1 and -1, respectively. The product of k successive tosses would be tabulated as the random binary variable. Products equal to +1 and -1 would be coded as 1 and 0, respectively. Blocks of binary symbols would then be converted to the decimal system as described above.

REFERENCE

- [1] TIPPETT, L. H. C., *Random Sampling Numbers*, Tracts for Computers, No. 15, Cambridge University Press, 1927.

NOTE ON THE ERROR IN INTERPOLATION OF A FUNCTION OF TWO INDEPENDENT VARIABLES

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Suppose that g is a function of one real variable x and h is an interpolation function such that $g(x) = h(x)$ for $x = x_1, x_2, \dots, x_n$. Let $f(x) = g(x) - h(x)$ and suppose that $\frac{d^n}{dx^n} f(x)$ exists in an interval containing the points x_0, x_1, \dots, x_n . Then the error in interpolation may be estimated from the well-known relation

$$(1) \quad f(x_0) = \frac{f^{(n)}(\xi)}{n!} (x_0 - x_1)(x_0 - x_2) \cdots (x_0 - x_n),$$

where ξ is some point in the smallest interval containing x_0, x_1, \dots, x_n .

In the most usual case, where $h(x)$ is a polynomial of degree less than n , we have $f^{(n)}(\xi) = g^{(n)}(\xi)$.

It is natural to consider the corresponding situation for functions of two independent real variables x and y . Let g and h be two functions such that $g(x, y) = h(x, y)$ for n points $x = x_i, y = y_i (i = 1, 2, \dots, n)$. Setting $f(x, y) = g(x, y) - h(x, y)$ as before, we have $f(x_i, y_i) = 0$ for $i = 1, 2, \dots, n$. Then if (x_0, y_0)