

Thus,

$$P(|\bar{Z}_n| \geq t) = 2[1 - P(\bar{Z}_n \leq t)] \\ = 2 \left\{ 1 - \frac{1}{n!} \sum_{i \leq (n/2)(t+1)} (-1)^i \binom{n}{i} \left[ \frac{n}{2} (t+1) - i \right]^n \right\},$$

and in view of the identity

$$\sum_{k=0}^n (-1)^k \binom{n}{k} (u - k)^n = n!$$

this becomes

$$P(|\bar{Z}_n| \geq t) = \frac{2}{n!} \sum_{(n/2)(t+1) < k \leq n} (-1)^k \binom{n}{k} \left[ \frac{n}{2} (t+1) - k \right]^n = \Psi_n(t)$$

for  $0 \leq t \leq 1$ . The random variable  $\frac{Y}{a}$  is obviously more peaked about zero than  $Z$ . Since  $\frac{Y}{a}$  and  $Z$  fulfil the assumptions of Theorem 1, it follows that  $\frac{\bar{Y}_n}{a}$  is more peaked about zero than  $\bar{Z}_n$ , that is

$$P\left(\left|\frac{\bar{Y}_n}{a}\right| \geq t\right) \leq P(|\bar{Z}_n| \geq t) = \Psi_n(t) \quad \text{for } t \geq 0.$$

Setting  $at = y$ , one obtains (4.1).

For  $n \rightarrow \infty$  the function  $\Psi_n(t)$  approaches asymptotically the probability  $P(|X| \geq t\sqrt{3n})$  for the normalized normal random variable  $X$ .<sup>4</sup> For  $n = 8$  one obtains the following values which indicate a good approximation:

$t$	.3998	.5254	.6711
$P( X  \geq t\sqrt{24})$	.05	.01	.001
$\Psi_8(t)$	.049	.0092	.0005.

For smaller values of  $n$ ,  $\Psi_n(t)$  can be easily computed.

## A METHOD FOR OBTAINING RANDOM NUMBERS

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The need for large quantities of random numbers to be used in sample design, subsampling, and other statistical problems is well known. Tippett's [1] numbers have been widely used for these purposes, despite criticism directed at their lack of randomness. The following procedure may be of interest to those

<sup>4</sup> Cramér, *op. cit.*, p. 245.