where the summations include the terms $E[x_i^2]$ and $E[x_{n-i+1}{}^2]$, respectively. But it is known [2] that the sample mean is the regular unbiased estimate of μ with minimum variance. Setting each w equal to $1/n$ and combining equivalent terms yields

$$\sum_{j=1}^{n} E[x_i x_j] + \tfrac{1}{2} n\lambda = 0, \qquad i = 1, 2, \cdots, n.$$

Summing from $i = 1$ to $i = n$, and employing the relationships discussed in the preceding paragraph, we obtain

$$n + \tfrac{1}{2} n^2 \lambda = 0,$$

whence

$$\lambda = -2/n,$$

and

$$\sum_{j=1}^{n} E[x_i x_j] = 1, \qquad i = 1, 2, \cdots, n,$$

where the summation includes the term $E[x_i^2]$. This equation leads to the properties mentioned at the beginning of this paragraph. The same equation can be used to evaluate $E[x_1^2]$ and $E[x_2^2]$ in samples of size 3 or 4 from the distribution $N(0, 1)$, after the product-moments have been found.

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NOTE ON AN ASYMPTOTIC EXPANSION OF THE nTH DIFFERENCE OF ZERO

By L. C. Hsu

National Tsing-Hua University, Peiping, China

This note gives an asymptotic expansion of the nth difference of zero. It is known that the Stirling number $S_{n,s}$ of the second kind is defined by

$$(1) \qquad n!\,S_{n,s} = \Delta^n 0^s = \sum_{x=0}^{n} (-1)^{n-x} \binom{n}{x} x^s.$$

We shall first show that the Stirling number $S_{n,n+k}$ can be expanded in the form

$$(2) \qquad S_{n,n+k} = \frac{n^{2k}}{2^k \cdot k!}\left[1 + \frac{f_1(k)}{n} + \frac{f_2(k)}{n^2} + \cdots + \frac{f_t(k)}{n^t} + O(n^{-t-1})\right], \quad (t < k)$$