NOTES

This section is devoted to brief research and expository articles and other short items.

A FUNCTIONAL EQUATION FOR WISHART'S DISTRIBUTION

By G. Rasch

State Serum Institute and University of Copenhagen

1. Introduction. The sampling distribution of the moment matrix for observations from a multivariate normal distribution was given by Wishart in 1928 [1]. This proof involved rather advanced multidimensional geometry but since then two analytical proofs have been given: one by Wishart and Bartlett in cooperation with Ingham by the use of the characteristic function [2] and a second by Hsu by induction with regard to the dimension of the observations, [3],

In the following section is given a new derivation of the form of Wishart's distribution in which a fundamental property of the multivariate normal distribution is utilized, *viz.* the invariance of the distribution type against a linear transformation. In section 3 the same principle is used for evaluation of the constant and determination of the moment matrix in the multidimensional normal distribution.

2. Derivation of Wishart's distribution. Let[1]

$$\mathbf{x} = (x_1, \cdots, x_k), \tag{1}$$

denote a k-dimensional normal variate with the mean vector 0 and the distribution matrix

$$\mathbf{\Phi} = (\varphi_{ij}), \tag{2}$$

viz.

$$p\{\mathbf{x}\} = \frac{\sqrt{\Delta(\mathbf{\Phi})}}{(\sqrt{2\pi})^k} \cdot e^{-\frac{1}{2}\mathbf{x}\mathbf{\Phi}\mathbf{x}^*}. \tag{3}$$

$\mathbf{\Phi}$ is symmetrical and positive definite.

Now consider n observations of $\mathbf{x}$: $\mathbf{x}_1, \cdots, \mathbf{x}_n$, which are stochastically independent. Their joint distribution is

$$p\{\mathbf{x}_1, \cdots \mathbf{x}_n\} = \left(\frac{\sqrt{\Delta(\mathbf{\Phi})}}{(\sqrt{2\pi})^k}\right)^n \cdot e^{-\frac{1}{2}\Sigma \mathbf{x}_\nu \mathbf{\Phi} \mathbf{x}_\nu^*}. \tag{4}$$

The estimation of $\mathbf{\Phi}$ is based upon the moment sums

$$m_{ij} = \Sigma x_{\nu i} x_{\nu j},$$

[1] Notations: Lower case latin and greek letters are scalars; boldface capital latin and greek letters denote matrices, and boldface lower case letters row vectors. * means transposition. Δ (**A**) stands for the determinant of the square matrix **A**.