to give:

$$\begin{split} G(r) &= 1 - \left\{ \frac{A - \pi r^2}{A} \right\}^{m-1}, \\ G'(r) &= \frac{2\pi r}{A} (m-1) \left\{ \frac{A - \pi r^2}{A} \right\}^{m-2}, \\ E(d) &= \int_0^\rho r G'(r) dr = \frac{1}{2} \sqrt{\frac{A}{\pi}} \left[B(m, \frac{1}{2}) \right], \end{split}$$

where $B(m, \frac{1}{2})$ is the complete Beta function. Since $\sqrt{m} [B(m, \frac{1}{2})] \ge \sqrt{\pi}$:

$$E(d) \geq \frac{1}{2} \sqrt{\frac{\overline{A}}{m}}$$

Thus, we have:

$$E(L) \geq \frac{1}{2} \sqrt{A} \frac{m-1}{\sqrt{m}}.$$

It is obvious that the development is general and applies to m random points in any bounded two-dimensional Borel set. However, the lower bound obtained will, in general, be useful only when S is a connected region.

REFERENCES

- [1] RAYMOND J. JESSEN, "Statistical investigation of a sample survey for obtaining farm facts," Iowa State College Research Bulletin 304 (1942).
- [2] P. C. Mahalanobis, "A sample survey of the acreage under jute in Bengal," Sankhyā, Vol. 4 (1940), pp. 511-530.

A MATRIX ARISING IN CORRELATION THEORY¹

By H. M. BACON

Stanford University

1. Introduction. In the study of time series, it is frequently desirable to consider correlations between observations made in different years. Let x_{ii} , x_{i2} , \cdots , x_{im} be m values of the variable x_i , expressed as deviations from their arithmetic mean, where x_i is a variable observed in the *i*th year $(i = 1, 2, \dots, n)$.

¹ A linear correlogram is considered by Cochran in his paper, "Relative accuracy of systematic and stratified random samples for a certain class of populations," (Annals of Math. Stat., Vol. 17 (1946), pp. 164–177) in which $\rho_{\mu} = 1 - \frac{\mu}{L}$. Setting $\mu = |i - j|$ and L = 1/p, we have the case considered above.