It remains to be shown that k as determined by (8) equals l. This will be so if we can show that

$$I_{\frac{1}{2}}(n - l + 1, l) \leq \frac{\alpha}{2} < I_{\frac{1}{2}}(n - l, l + 1). \tag{11}$$

Remembering that $I_x(p, q)$ is a monotonically increasing function of x we get with the help of (7) and (10)

$$\frac{\alpha}{2} = I_{1-\theta_{l-1}}(n - l + 1, l) \geq I_{\frac{1}{2}}(n - l + 1, l)$$

and

$$\frac{\alpha}{2} = I_{1-\theta_l}(n - l, l + 1) < I_{\frac{1}{2}}(n - l, l + 1)$$

which proves (11).

In conclusion it may be worth while pointing out that the formula

$$P\{Z_i < q_p < Z_j\} = I_p(i, n - i + 1) - I_p(j, n - j + 1)$$

given, e.g, in Wilks [1] for the continuous case can be obtained by a slight modification of (6).

REFERENCES

[1] S. S. Wilks, "Order statistics," *Am. Math. Soc. Bull.*, Vol. 54 (1948), pp. 6–50.
[2] H. Scheffé and J. W. Tukey, "Non-parametric estimation. I. Validation of order statistics," *Annals of Math. Stat.*, Vol. 16 (1945), pp. 187–192.
[3] C. J. Clopper and E. S. Pearson, "The use of confidence or fiducial limits illustrated in the case of the binomial," *Biometrika*, Vol. 26 (1934), pp. 404–413.
[4] H. Scheffé, "Note on the use of the tables of percentage points of the incomplete beta function to calculate small sample confidence intervals for binomial p," *Biometrika*, Vol. 33 (1944), p. 181.
[5] C. M. Thompson, E. S. Pearson, L. J. Comrie, and H. O. Hartley, "Tables of percentage points of the incomplete beta function," *Biometrika*, Vol. 32 (1941), pp. 151–181.
[6] W. R. Thompson, "On confidence ranges for the median and other expectation distributions for populations of unknown distribution form," *Annals of Math. Stat.*. Vol. 7 (1936), pp. 122–128.

A LOWER BOUND FOR THE EXPECTED TRAVEL AMONG m RANDOM POINTS

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In connection with cost determinations in sampling problems, it is frequently necessary to determine the amount of travel among m random sample points in an area. A lower bound for the expected value of this distance is found to be:

$$\sqrt{\frac{A}{2}}\frac{m-1}{\sqrt{m}},$$