where $\phi(z)$ is the ordinary m.g.f. of a non-negative random variable. Likewise a necessary and sufficient condition for $\omega(z)$ to be the f.m.g.f. of a generalized Poisson distribution is that it be of the form

$$\omega_2(z) = e^{\alpha(\Omega(z)-1)}, \qquad \alpha > 0, \tag{2}$$

where $\Omega(z)$ is the f.m.g.f. of an arbitrary distribution function $F(x)$. If we choose $\phi(z) = e^{\alpha(e^{cz}-1)}$ and $\Omega(z) = e^{cz}$, then $\omega_1(z) = \omega_2(z)$, and the distribution whose f.m.g.f. is $\omega_1(z)$ (the Neyman contagious distribution of Type A) is simultaneously a compound and a generalized Poisson distribution (cf. Feller [2]). We now show that there is an infinite class of distributions with this property.

First note that if $\phi(z)$ is the m.g.f. of an arbitrary distribution, then exp $\{\alpha(\phi(z) - 1)\}$ is also the m.g.f. of a d.f., and in fact is the m.g.f. of the generalized Poisson distribution associated with the distribution whose m.g.f. is $\phi(z)$. Now let $\phi(z)$ be the m.g.f. of an arbitrary non-negative random variable, and define

$$\omega(z) = \exp\{\alpha(\phi(z) - 1)\} \qquad \alpha > 0. \tag{3}$$

Then $\omega(z)$ is simultaneously of the forms (1) and (2), since $\phi(z)$ is, by (1), also the f.m.g.f. of a distribution function, i.e. the compound Poisson distribution associated with the distribution whose m.g.f. is $\phi(z)$. However, not every distribution which is both a compound and a generalized Poisson distribution can be generated in this manner. For example, the Polya-Eggenberger distribution is easily shown to be both a generalized and a compound Poisson distribution, yet its f.m.g.f.

$$\omega(z) = (1 - dz)^{-h/d}, \qquad d > 0, h > 0,$$

manifestly is not of the form (3), since this would imply $\phi(iz) = 1 - \frac{h}{\alpha d} \log (1 - diz)$ is a characteristic function. But $|\phi(iz)|$ is unbounded as $z \to \pm\infty$ and thus is not the characteristic function of a distribution.

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ON CONFIDENCE LIMITS FOR QUANTILES

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In finding confidence limits for quantiles it is usual to determine two order statistics Z_i and Z_j which with a given probability contain the unknown quantile