NOTES

This section is devoted to brief research and expository articles and other short items.

THE DISTRIBUTION OF STUDENT'S t WHEN THE POPULATION MEANS ARE UNEQUAL

By Herbert Robbins

Department of Mathematical Statistics, Universtiy of North Carolina

Let $x_1, \cdots, x_N$ be independent normal variates with the same variance σ^2 and with means $\mu_1, \cdots, \mu_N$ respectively. Set $n = N - 1$ and let

$$(1) \qquad \bar{x} = \sum_1^N x_i/N, \qquad s^2 = \sum_1^N (x_i - \bar{x})^2/n, \qquad t = N^{\frac{1}{2}}\bar{x}/s.$$

If all the μ_i are 0 then t has Student's distribution with n degrees of freedom; its frequency function will be denoted here by

$$(2) \qquad f_{n,0}(t) = n^{-\frac{1}{2}}\left[B\left(\frac{1}{2}, \frac{n}{2}\right)\right]^{-1} \cdot (1 + t^2/n)^{-\frac{1}{2}(n+1)}.$$

When dealing with situations involving mixtures of populations or in which the mean exhibits a secular trend, it is important to know the distribution of t when the μ_i are arbitrary; in the general case let

$$(3) \qquad \begin{aligned} \bar{\mu} &= \sum_1^N \mu_i/N, \qquad \beta^2 = \sum_1^N (\mu_i - \bar{\mu})^2/N, \\ \alpha &= N\bar{\mu}^2/2\sigma^2, \qquad \lambda = N\beta^2/2\sigma^2. \end{aligned}$$

The distribution of t will be shown to depend on the three parameters n, α, λ. If $\lambda = \beta^2 = 0$, so that all the μ_i are equal, then the distribution of t determines the power function of the ordinary t test. We shall here consider the case in which $\alpha = \bar{\mu} = 0$, although the μ_i are different. Denoting the frequency function of t in this case by $f_{n,\lambda}(t)$ we shall show that

$$(4) \qquad f_{n,\lambda}(t) = f_{n,0}(t) \cdot \exp\left\{\frac{-\lambda t^2/n}{1 + t^2/n}\right\} \cdot F(-\tfrac{1}{2}, n/2, -\lambda(1 + t^2/n)^{-1}),$$

where F denotes the confluent hypergeometric series, and where, since $\bar{\mu} = 0$,

$$(5) \qquad \lambda = \sum_1^N \mu_i^2/2\sigma^2.$$

In fact, the general distribution of t, of which (4) represents the case $\alpha = 0$,