BOUNDS FOR SOME FUNCTIONS USED IN SEQUENTIALLY TESTING THE MEAN OF A POISSON DISTRIBUTION[1]

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1. Introduction. Let $z = \log \frac{f(x, \lambda_1)}{f(x, \lambda_0)}$, where $f(x, \lambda_i) = (e^{-\lambda_i} \lambda_i^x)/x!$, $(i = 0, 1)$, is the elementary probability law of a Poisson variate X, under the hypothesis that the mean is equal to λ_i. Without loss of generality we shall assume $\lambda_1 > \lambda_0$.

Let H_0 be the hypothesis that the distribution of X is given by $f(x, \lambda_0)$. Wald [1, pp. 286–287] has devised general upper and lower bounds for the probability of accepting H_0, when λ is the true value of the parameter, and the sequential probability ratio test is used. This probability is called the operating-characteristic function and is designated by $L(\lambda)$. Using these results he has computed the bounds for the binomial and normal distributions [2, pp. 137–142]. We shall do the same thing for the Poisson distribution, since the restrictions [1, p. 284, conditions I to III] under which these general limits are valid can rather easily be shown to apply to the Poisson distribution, if we make the further restriction that $E(z) \neq 0$.

These general results are

$$\frac{1 - B^h}{\delta A^h - B^h} \leq 1 - L(\lambda) \leq \frac{1 - \eta B^h}{A^h - \eta B^h}, \qquad \text{if } h > 0,$$

and

$$\frac{1 - A^h}{\delta B^h - A^h} \leq L(\lambda) \leq \frac{1 - \eta A^h}{B^h - \eta A^h}, \qquad \text{if } h < 0, \tag{1}$$

where α, β are probabilities of committing errors of the first and second kind respectively and

$$\text{A} = (1 - \beta)/\alpha, \qquad \text{B} = \beta/(1 - \alpha)$$

$$\eta = \operatorname*{glb}_{\zeta} \zeta E\left(e^{hz} \mid e^{hz} < \frac{1}{\zeta}\right), \qquad \zeta > 1;$$

$$\delta = \operatorname*{lub}_{\rho} \rho E\left(e^{hz} \mid e^{hz} \geq \frac{1}{\rho}\right), \qquad 0 < \rho < 1; \tag{2}$$

and h is the non-zero root of the expression, $Ee^{zt} = 1$. Hence the only remaining unknowns are η and δ.

[1] The author is indebted to Professor A. Wald for suggesting the problem which led to this note and for helpful discussions.