WEIGHING DESIGNS AND BALANCED INCOMPLETE BLOCKS

By K. S. Banerjee

Pusa, Bihar, India

1. Introduction. Following a paper by Hotelling [1] on the weighing problem, Kishen [4] and Mood [2] furnished generalized solutions. This note consists of some additional remarks on the weighing problem when the weighing is restricted to be made on one pan.

Hotelling remarked that when the problem was to determine a particular difference or any other linear function of the weights, a different design should be sought to minimize the variance. An account of efficient designs of this kind has also been furnished in this note. The notations used by Hotelling and Mood have been used here.

2. Chemical balance problem. It has been shown by Mood that when $N \equiv 0 \pmod 4$, an optimum design exists if a Hadamard matrix H_N exists, and is obtained by using any p columns of H_N. When $N \equiv i \pmod 4$, $(i = 1, 2, 3)$, very efficient designs are obtained either by adding to or deleting from the rows of H_{4K}, making the resultant number of rows equal to N.

It has further been shown by Mood in connection with this class of designs that arrangements[1] are available which are more efficient than the one obtained by repeating the row of ones. As a matter of fact, if any row other than the row of ones be repeated, this will lead to a design of the same efficiency as in the case of repeated addition of the row of ones; for, the determinant of $X'X$ will remain exactly identical. That this is so, will be clear from the following properties showing the connection of the matrix X with the determinant $| a_{ij} |$:

(i) Any two rows of the matrix X can be interchanged without changing the determinant $| a_{ij} |$.

(ii) Any two columns of the matrix X can be interchanged without changing the determinant $| a_{ij} |$.

(iii) The signs of all the elements in a column of the matrix X may be changed without changing the determinant $| a_{ij} |$.

3. Spring balance problem. Mood has exhaustively discussed the designs when $N > p$. Efficient designs under this class will, however, be available from the arrangements afforded by balanced incomplete block designs discussed in [3]. These designs will be represented by certain of the efficient submatrices of the P_k of Mood.

Usually v and b are used to denote respectively the number of varieties and the number of blocks in the above mentioned designs. Here v will take the place of

[1] This had been independently shown by me before the paper of A. M. Mood was brought to my notice by H. Hotelling.