ABSTRACTS OF PAPERS

Presented at the Madison Meeting of the Institute, September 7–10, 1948

1. On Distribution-free Confidence Intervals (Preliminary Report). WASSILY HOEFFDING, University of North Carolina, Chapel Hill.

Let $\theta(F)$ be a functional of a distribution function (d.f.) $F(x)$ (where x is a real number or a vector), defined over a class $\mathfrak{D}$ of d.f.'s; O_n a random sample from a population with d.f. $F(x)$; $\underline{\theta}_n \leq \bar{\theta}_n$ two functions of O_n; and $\alpha_n = Pr\{\underline{\theta}_n \leq \theta(F) \leq \bar{\theta}_n\}$. Conditions are studied under which, given α, $0 < \alpha < 1$, we have either $\alpha_n = \alpha$ or $\alpha_n \geq \alpha$ or $\alpha_n \to \alpha$, for all $F(x)$ in $\mathfrak{D}$, where $\mathfrak{D}$ is defined independently of the functional form of $F(x)$. Under fairly general conditions we can obtain by "studentization" confidence limits $\underline{\theta}_n$, $\bar{\theta}_n$ such that $\lim_{n\to\infty} \alpha_n = \alpha$, and $\gamma = \lim_{n\to\infty} E\sqrt{n}(\bar{\theta}_n - \underline{\theta}_n)$ exists; γ is minimized by using a least variance estimate of $\theta(F)$. If there exists a function $\kappa(\theta)$ such that var $T_n \leq \kappa^2(\theta)n^{-1}$ if $\theta(F) = \theta$, for all F in $\mathfrak{D}$, we can define confidence limits with a positive lower bound for α_n. This applies to a number of population characteristics estimated by rank order statistics, such as the coefficients ρ' and τ (estimated by Spearman's and Lindeberg-Kendall's rank correlation coefficients, respectively). In certain cases (including ρ' and τ), $\theta(F)$ admits a binomially distributed estimate; then exact confidence limits can easily be obtained. This research was done under an Office of Naval Research contract.

2. On Certain Statistics for Samples of 3 from a Normal Population. JULIUS LIEBLEIN, National Bureau of Standards, Washington.

In analytical chemistry three determinations are frequently made. Sometimes the average of only the two *closest* results is reported, the remaining observation being rejected as anomalous. In preparing a critique of this procedure, Dr. W. J. Youden encountered a need for information on certain properties of the distributions of the statistics $(x' - x'')/(x_3 - x_1)$, $(x' + x'')/2$, and $(x' - x'')/2$, where x' and x'' $(x' \geq x'')$ are the two *closest* of the three determinations. This paper shows how these statistics differ from the ones heretofore treated involving "fixed" order statistics; gives the distribution of these statistics in random samples of 3 from a normal universe; and lists values of certain of the moments of their distributions.

3. On Multinomial Distributions with Limited Freedom: A Stochastic Genesis of Pareto's and Pearson's Curves. MARIA CASTELLAIN, University of Kansas City.

The purpose of this paper is to investigate the most probable configuration of N random elements to be distributed in $K(K < N)$ class intervals, where known forces are acting. We shall call these intervals of energy, using the terminology of statistical mechanics.

We will prove that the most probable configuration is a configuration of statistical equilibrium since its probability of occurring converges to 1 as N becomes infinitely large.

The main purpose of this paper is to discover which forces of attraction, operating in the intervals of energy, give Pareto's and Pearson's curves when statistical equilibrium is reached.

We will consider a random variable $Y(t)$, t being an independent variable, obeying a multinomial distribution law with limited freedom, and we will exploit the familiar process of statistical mechanics. The equation of the frequency curves corresponding to the equilibrium stage of the statistical experiment will be shown.