[2] A. L. O'Toole, "On the system of curves for which the method of moments is the best method of fitting," *Annals of Math. Stat.*, Vol. 4 (1933), pp. 1–29.
[3] A. L. O'Toole, "A method of determining the constants in the bimodal fourth degree exponential function," *Annals of Math. Stat.*, Vol. 4 (1933), pp. 79–93.
[4] J. Ernest Wilkins, Jr., "A note on skewness and kurtosis," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 333–335.
[5] C. V. L. Charlier, "A new form of the frequency function," *Lund universitet, Acta*, N. F. Bd 24 (1928), Avd. 2, Art. no. 8, pp. 1–26.
[6] M. G. Kendall, *The Advanced Theory of Statistics*, Griffin & Co., London, Vol. II, p. 43.

AN APPROXIMATION TO THE BINOMIAL SUMMATION

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We consider the binomial expansion $(q + p)^n$, where $q = 1 - p$ and n is a positive integer. For given values of n, p, r, and s, where $np < r < s \leq n$, we are often interested in the probability $P(r \leq x \leq s)$ that the number of successes x will satisfy $r \leq x \leq s$.

When n does not exceed 50, we can use tables of the Incomplete Beta Function, or other convenient and accurate tables. For "large" values of n, we can use normal tables. When p is "small", we can use Poisson tables. However, it is often true that p is fairly small, and yet not small enough to give really accurate results when Poisson tables are employed in the usual way, while n is too large for use of the tables of the Incomplete Beta Function and yet too small for accurate use of normal tables.

It frequently happens that an upper bound for $P(r \leq x \leq s)$ would serve our purpose. We propose to show how to find this from Poisson tables with greater accuracy than could be obtained by using these tables in the ordinary way.

We shall denote the general term of the binomial expansion by $B_i = \binom{n}{i} p^i q^{n-i}$ and the general term of the corresponding Poisson distribution with the same value of p by $P_i = (pn)^i e^{-pn}/i!$. We shall also consider a second Poisson distribution whose general term is given by $P'_i = (p'n)^i e^{-p'n}/i!$, where $p' \neq p$ will be determined later.

We shall use the following notations:

$$U_i = B_{i+1}/B_i = (n - i)(p)/(i + 1)(1 - p); \tag{1}$$

$$V_i = P_{i+1}/P_i = pn/(i + 1); \tag{2}$$

$$V'_i = P'_{i+1}/P'_i = p'n/(i + 1); \tag{3}$$

$$U_i - V_i = p(np - i)/(i + 1)(1 - p). \tag{4}$$

From (4) we obtain at once the following:

Lemma I. *$U_i > V_i$ or $U_i < V_i$ according as $i < np$ or $i > np$.*

Thus, the size of the general term of the binomial expansion falls off more steeply to the right of $i = np$ than does that of the general Poisson term.