ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Seattle Meeting of the Institute on November 26–27, 1948)

1. Estimation of the Variance of the Bivariate Normal Distribution. HARRY M. HUGHES, University of California, Berkeley.

Let x_1 and x_2 be two random variables normally distributed with known means m_1 and m_2, and with common unknown variance σ^2. Consider an experiment in which the observed variable is $Y = \sqrt{(x_1 - m_1)^2 + (x_2 - m_2)^2}$. This paper considers the problem of estimating the parameter σ when the observations are grouped. By the method of minimum reduced chi-square with linear restrictions, two best asymptotically normal estimates are derived. By minimization of the asymptotic variance of these estimates, the optimum choice of grouping is found as a function of σ. For two and for three groups, when it is known or assumed a priori that σ is on a certain finite interval, the optimum grouping is derived which will minimize the maximum asymptotic variance on that interval. If such interval is moderately small, it is shown that the optimum grouping is the same as if σ were known to have the value at the upper end of the interval. Finally the effect of using non-optimum grouping is analyzed.

2. Derivation of a Broad Class of Consistent Estimates. R. C. DAVIS, Inyokern, California.

Given a chance vextor X with cumulative distribution function $F(X, \theta)$, where θ is an unknown parameter vector, a broad class of estimates of θ is derived having the following properties: a) any estimate in this class is a consistent estimate of θ; b) any estimate is a symmetric function of independent observations of the chance vector X. The novel feature of this class is that no assumptions about the existence of various partial derivatives of a density function with respect to θ are made. As a matter of fact not even the existence of a density function is required, and it is postulated merely that a continuous function of X for each θ (in a certain neighborhood of the true θ_0) and of θ for each X exist which satisfies a Lipschitz condition in θ. For each such function having a finite first moment an estimate of θ is constructed which has the properties a) and b) listed above.

3. Locally Best Unbiased Estimates. EDWARD W. BARANKIN, University of California, Berkeley.

Let $p = \{p_\vartheta(x); \vartheta \in \Theta\}$ be a family of probability densities in the space Ω of points x; and g a function on Θ. Let s be fixed and >1; call an unbiased estimate of g *best at* ϑ_0 if its s-th absolute central moment (s.a.c.m.) under p_{ϑ_0} is (finite and) not greater than the s.a.c.m., under p_{ϑ_0}, of any unbiased estimate of g. With a certain integrability postulate on the p_ϑ's, a necessary and sufficient condition, of finite character, is established for the existence of an unbiased estimate of g having a finite s.a.c.m. under p_{ϑ_0}. When such a one exists, there then exists a *unique* unbiased estimate which is best at ϑ_0. The existence condition defines the s.a.c.m. of the best estimate explicitly as the l.u.b. of a set of numbers; in particular, we obtain immediately a generalization of the Cramér-Rao inequality. Also, when it exists, the best unbiased estimate is explicitly constructed from the function g and the densities p_ϑ. The case $s = 2$ is studied more closely. Also, a detailed example is considered.