AN APPROXIMATION TO THE SAMPLING VARIANCE OF AN ESTIMATED MAXIMUM VALUE OF GIVEN FREQUENCY BASED ON FIT OF DOUBLY EXPONENTIAL DISTRIBUTION OF MAXIMUM VALUES[1]

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1. Introduction. Given the doubly exponential distribution of maximum values

(1) $$F(x) = \exp(-e^{-y}), \qquad y = \alpha(x - u),$$

where α and u are unknown parameters, with a prescribed frequency F_0 the "reduced variate" y is fixed, say at $y = y_0$. Thus with

$$F_0 = .99, \qquad y_0 = 4.60015 \cdots .$$

Given a sample of n maximum values x_i, we are interested in the sampling variance of

(2) $$\hat{x} = g(\hat{u}, \hat{\alpha}) = \hat{u} + y_0/\hat{\alpha}$$

due to sampling variations of the estimates $\hat{u}$ and $\hat{\alpha}$.

H. Fairfield Smith has recently pointed out to me that the examples of applications of sufficient statistical estimation functions to this problem given in a previous paper (see [1, pp. 307–309]) give too large a range for $\hat{x} = g(\hat{u}, \hat{\alpha})$ because the sample points $(\hat{u}, \hat{\alpha})$ within the confidence region of the constant probability ellipse apply to optimum estimates of $(\hat{u}, \hat{\alpha})$ rather than to that of $g = g(\hat{u}, \hat{\alpha})$. What the problem calls for is the determination of the positions of curves $\bar{g}(u, \alpha)$ and $\underline{g}(u, \alpha)$ such that the integral of the *pdf* of the estimation functions over all sample values $(\hat{u}, \hat{\alpha})$ which lie between these two curves is equal to the confidence level (taken as .95 in previous paper). Further considerations of this being the shortest interval $\bar{g} - \underline{g}$, also come into play.

As so often happens in research, the previous analysis, although not giving the final answer, suggests the next step. If we change our parameters to

(3) $$g = g(u, \alpha) = u + y_0/\alpha, \qquad \alpha' = \alpha$$

and are able to carry through the inverse of the maximum likelihood solution for fitting of (1) to n sample values x_i, then we shall be in a position to find the asymptotic marginal distribution of $\sqrt{n}(\hat{g} - g)$, which will give the answer to our problem (see [2]).

The Jacobian of this transformation of parameters is

$$\partial(u, \alpha)/\partial(g, \alpha') = \begin{vmatrix} 1 & y_0^2/\alpha'^2 \\ 0 & 1 \end{vmatrix} = 1,$$

and hence for $\alpha' > 0$ no new singularities are introduced.

[1] This involves a correction of a previous paper [1].