NOTES

This section is devoted to brief research and expository articles on methology and other short items.

BROWNIAN MOTION ON THE SURFACE OF THE 3-SPHERE

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1. Introduction. Let S be a n-dimensional compact riemann space with the metric $ds^2 = g_{ij}(x)\, dx^i\, dx^j$ such that the totality G of the isometric transformations of S onto S constitutes a Lie group transitive on S. Consider a temporally homogeneous Markoff process by which $P(t, x, y)$, $t > 0$, is the transition probability that a point x is transferred to y after the elapse of t-unit time. We assume that $P(t, x, y)$ is a Baire function in (t, x, y) and continuous in t, then P satisfies Smoluchouski's equation

$$P(t + s, x, y) = \int_S P(t, x, z)P(s, z, y)\, dz \qquad (t, s > 0), \tag{1.1}$$

dz being the G-invariant measure $\sqrt{g(x)}dx^1\, dx^2 \cdots dx^n$, $g(x) = \det(g_{ij}(x))$, and

$$P(t, x, y) \geqq 0, \tag{1.2}$$

$$\int_S P(t, x, y)\, dy = 1. \tag{1.3}$$

The spatial homogeneity of the transition process may be defined by

$$P(t, Tx, Ty) = P(t, x, y) \qquad \text{for } T \in G. \tag{1.4}$$

The "continuity" of the transition process may be defined, following after A. Kolmogoroff and W. Feller,[1] as follows. Let $L_1(S)$ be the function space of integrable (with respect to dx) functions $f(x)$ on S, then, for those $f(x)$ which are dense in $L_1(S)$,

$$\begin{aligned} &\frac{\partial f(t, x)}{\partial t} = A \cdot f(t, x), \qquad (t \geqq 0); \\ &f(t, x) = \int_S f(y)P(t, y, x)\, dy, \qquad (t > 0), \qquad f(0, x) = f(x), \end{aligned} \tag{1.5}$$

where, with non-negative $b^{ii}(x)$

$$(Af)(x) = \frac{1}{\sqrt{g(x)}} \frac{\partial}{\partial x^i} (-\sqrt{g(x)}\, a^i(x)f(x)) + \frac{1}{\sqrt{g(x)}} \frac{\partial^2}{\partial x^i\, \partial x^j} (\sqrt{g(x)}\, b^{ij}(x)f(x)). \tag{1.6}$$

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[1] A. Kolmogoroff, "Zur Theorie der stetigen zufälligen Prozesse," *Math. Annalen*, Vol. 108 (1933); W. Feller, "Zur Theorie der stochastischen Prozesse," *Math. Annalen*, Vol. 113 (1937).